CHAPTER 3: REVIEW

Understanding: Concepts, Definitions, Formulas

Refer to the listed pages to review the concepts, definitions, and formulas in this chapter that you need to understand.

Section		Pages
3.1	The definition of the derivative	106
	The derivative as a slope predictor	106
	Differential notation for derivatives	108
	Average and instantaneous rate of change of a function	109–110, 114
	Position function; velocity and acceleration	112–114
3.2	Operator notation for derivatives	119
	The power rule: $D_x x^n = nx^{n-1}$	120, 126, 139
	Linearity of differentiation: $D_x(au + bv) = au' + bv'$	121
	The derivative of a polynomial	123
	The product rule: $D_x(uv) = u'v + uv'$	124
	The reciprocal rule	125
	The quotient rule: $D_x \dfrac{u}{v} = \dfrac{u'v - uv'}{v^2}$	126
3.3	The chain rule in differential notation: If $y = u(x)$ then $\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$	131
	The chain rule in functional notation: $D_x f(g(x)) = f'(g(x))g'(x)$	132
	The generalized power rule: $D_x u^n = nu^{n-1}\dfrac{du}{dx}$	134, 140
3.4	The definition of a vertical tangent line	142
	The continuity of differentiable functions	143
3.5	Maximum and minimum values of a function on a closed interval	147
	The maximum-minimum value property for continuous functions	147
	The necessary condition $f'(x) = 0$ for a local extreme value	148
	Local and global (absolute) extreme values	148–149
	The definition of a critical point	149
	The closed-interval maximum-minimum method	149–150
3.6	Steps in the solution of an applied maximum-minimum problem	156
3.7	The sine-cosine derivatives: $D_x \sin x = \cos x$, $D_x \cos x = -\sin x$	170
	The tangent-cotangent derivatives: $D_x \tan x = \sec^2 x$, $D_x \cot x = -\csc^2 x$	172
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	Chain rule forms of the trigonometric differentiation formulas	173
3.8	The general exponential function a^x and the laws of exponents	180–181
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	The natural exponential function e^x; its derivative $De^x = e^x$	184
	The chain-rule exponential derivative: $D_x e^u = e^u D_x u$	184
	The general logarithm function $\log_a x$ and laws of logarithms	185, 187
	Pairs of inverse functions	186
	Differentiation of inverse functions	187
	The natural logarithm function $\ln x$; its derivative $D_x \ln x = \dfrac{1}{x}$	188–189
	The chain-rule logarithmic derivative: $D_x \ln \lvert u \rvert = \dfrac{1}{u}\dfrac{du}{dx}$	190
	The process of logarithmic differentiation	190
3.9	Implicitly defined functions and implicit differentiation	194–195
	Related-rates problems and derivatives of related functions	197
3.10	~~Iteration and the Babylonian square root method~~	~~205–206~~
	~~Convergence of approximations to a solution of the equation $f(x) = 0$~~	~~207~~
	The iterative formula of Newton's method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$	207

CHAPTER 3: REVIEW (Continued)

Objectives: Methods and Techniques

Work the listed problems in each section to practice the methods and techniques in this chapter that you need to master.

Section		Problems
3.1	Using a differentiation rule to differentiate quadratic functions	5, 9
	Applying the definition of the derivative to find $f'(x)$	13, 17, 19
	Finding when the velocity of a moving particle is zero	25, 29, 39
	Matching the graphs of a function and its derivative	31, 33
	Calculating the rate of growth of a population	41, 53
	Calculating rates of change in geometric situations	45, 47, 49
3.2	Applying general differentiation rules to find derivatives	3, 5, 9, 11, 15, 19, 21, 27, 35
	Finding tangent lines to graphs	43, 45, 49
	Calculating rates of change in geometric situations	51, 53
3.3	Using the chain rule to differentiate functions	3, 5, 9, 13, 15, 23, 25, 29, 33, 35
	Calculating rates of change in geometric situations	49, 51, 53, 57, 59
3.4	Using rules to differentiate algebraic functions	3, 5, 9, 13, 17, 21, 23, 29, 35, 41
	Finding tangent lines to graphs of algebraic functions	47, 49, 53
	Matching the graphs of a function and its derivative	57, 59
3.5	Finding the maximum and minimum values of a function defined on a closed interval	5, 7, 11, 15, 19, 25, 33, 35, 37
	Matching the graphs of a function and its derivative	49, 51
3.6	Solving applied maximum-minimum problems	3, 5, 7, 11, 17, 21, 23, 27, 31, 33, 45
3.7	Calculating derivatives of trigonometric functions	5, 7, 9, 13, 15, 21, 27, 35, 39, 45, 47, 51, 53
	Finding tangent lines to trigonometric graphs	61, 65
	Solving trigonometric rate of change problems	75, 77
	Solving trigonometric maximum-minimum problems	79, 81, 83
3.8	Calculating derivatives of exponential and logarithmic functions	5, 7, 11, 15, 19, 23, 27, 29, 31, 33
	Applying laws of logarithms before differentiating	39, 41
	Finding a derivative by logarithmic differentiation	49, 51
	Finding tangent lines to exponential graphs	59, 61
3.9	Finding derivatives and tangent lines by implicit differentiation	3, 7, 15, 21, 25
	Solving applied related-rates problems	37, 39, 41, 43, 45, 47, 51, 53, 55, 61
3.10	Applying Newton's method to find a solution of an equation	3, 5, 9, 15, 17, 27, 33

MISCELLANEOUS PROBLEMS

Find dy/dx in Problems 1 through 35.

1. $y = x^2 + \dfrac{3}{x^2}$

2. $y^2 = x^2$

3. $y = \sqrt{x} + \dfrac{1}{\sqrt[3]{x}}$

4. $y = (x^2 + 4x)^{5/2}$

5. $y = (x-1)^7(3x+2)^9$

6. $y = \dfrac{x^4 + x^2}{x^2 + x + 1}$

7. $y = \left(3x - \dfrac{1}{2x^2}\right)^4$

8. $y = x^{10} \sin 10x$

9. $xy = 9$

10. $y = \sqrt{\dfrac{1}{5x^6}}$

11. $y = \dfrac{1}{\sqrt{(x^3 - x)^3}}$

12. $y = \sqrt[3]{2x+1}\,\sqrt[5]{3x-2}$

13. $y = \dfrac{1}{1+u^2}$ where $u = \dfrac{1}{1+x^2}$

14. $x^3 = \sin^2 y$

15. $y = \left(\sqrt{x} + \sqrt[3]{2x}\right)^{7/3}$

16. $y = \sqrt{3x^5 - 4x^2}$

17. $y = \dfrac{u+1}{u-1}$, where $u = \sqrt{x+1}$

18. $y = \sin(2\cos 3x)$

19. $x^2y^2 = x + y$

20. $y = \sqrt{1 + \sin\sqrt{x}}$

21. $y = \sqrt{x + \sqrt{2x + \sqrt{3x}}}$

CHAPTER 4: REVIEW (Continued)

Objectives: Methods and Techniques

Work the listed problems in each section to practice the methods and techniques in this chapter that you need to master.

Section		Problems
4.2	~~Calculating differentials of functions~~	~~1, 5, 9, 13~~
	Finding linear approximations to functions	17, 23
	Calculating numerical linear approximations	25, 31, 33
	~~Applying differentials in geometric situations~~	~~41, 43, 49~~
4.3	Using increasing-decreasing behavior to match functions and graphs	1, 3
	Determining the increasing-decreasing intervals for a function	11, 13, 19, 21
	Checking hypotheses and conclusions for Rolle's theorem	27, 31
	Checking hypotheses and conclusions for the mean value theorem	33, 35
4.4	Using the first derivative test to classify critical points	3, 7, 13, 21, 23
	Solving applied open-interval optimization problems	31, 33, 35, 41, 45
4.5	Using behavior at infinity to match functions and graphs	1, 3
	Finding critical points and increasing-decreasing behavior	7, 11
	~~Sketching graphs of given polynomials~~	~~15, 19, 23, 27~~
4.6	Calculating higher derivatives	3, 13, 17
	Finding critical and inflection points	23, 27
	Applying the second derivative and inflection point tests	33, 35, 47
	Using concavity and critical-inflection points to sketch graphs	63, 67, 75
	Matching graphs of functions and of their second derivatives	77, 79
4.7	Investigating infinite limits and limits at infinity	1, 3, 9
	Using asymptotes to match functions and their graphs	19, 21, 25
	Sketching graphs with extrema, inflection points, and asymptotes	35, 39, 43, 47, 49
4.8	Applying l'Hôpital's rule to the forms $0/0$ and ∞/∞	3, 9, 13, 19, 25, 29, 33
4.9	Applying l'Hôpital's rule to the forms $0 \cdot \infty$ and $\infty - \infty$	1, 7, 9, 13, 17
	Applying l'Hôpital's rule to the forms 0^0, ∞^0, and 1^∞	21, 23, 31

MISCELLANEOUS PROBLEMS

In Problems 1 through 6, write dy in terms of x and dx.

1. $y = (4x - x^2)^{3/2}$

2. $y = 8x^3\sqrt{x^2+9}$

3. $y = \dfrac{x+1}{x-1}$

4. $y = \sin x^2$

5. $y = x^2 \cos \sqrt{x}$

6. $y = \dfrac{x}{\sin 2x}$

In Problems 7 through 16, estimate the indicated number by linear approximation.

7. $\sqrt{6401}$ (Note that $80^2 = 6400$.)

8. $\dfrac{1}{1.000007}$

9. $(2.0003)^{10}$ (Note that $2^{10} = 1024$.)

10. $\sqrt[3]{999}$ (Note that $10^3 = 1000$.)

11. $\sqrt[3]{1005}$

12. $\sqrt[3]{62}$

13. $26^{3/2}$

14. $\sqrt[5]{30}$

15. $\sqrt[4]{17}$

16. $\sqrt[10]{1000}$

In Problems 17 through 22, estimate by linear approximation the change in the indicated quantity.

17. The volume $V = s^3$ of a cube, if its side length s is increased from 5 in. to 5.1 in.

18. The area $A = \pi r^2$ of a circle, if its radius r is decreased from 10 cm to 9.8 cm.

19. The volume $V = \frac{4}{3}\pi r^3$ of a sphere, if its radius r is increased from 5 cm to 5.1 cm.

20. The volume $V = 1000/p$ in.^{3} of a gas, if the pressure p is decreased from 100 lb/in.^{2} to 99 lb/in.^{2}

21. The period of oscillation $T = 2\pi\sqrt{L/32}$ of a pendulum, if its length L is increased from 2 ft to 2 ft 1 in. (Time T is in seconds and L is in feet.)

22. The lifetime $L = 10^{30}/E^{13}$ of a light bulb with applied voltage E volts (V), if the voltage is increased from 110 V to 111 V. Compare your result with the exact change in the function L.

If the mean value theorem applies to the function f on the interval $[a, b]$, it ensures the existence of a solution c in the interval (a, b) of the equation

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In Problems 23 through 28, a function f and an interval $[a, b]$ are given. Verify that the hypotheses of the mean value theorem are satisfied for f on $[a, b]$. Then use the given equation to find the value of the number c.

23. $f(x) = x - \dfrac{1}{x}$; $[1, 3]$

24. $f(x) = x^3 + x - 4$; $[-2, 3]$

CHAPTER 4: REVIEW

Understanding: Concepts, Definitions, Results

Refer to the listed pages to review the concepts, definitions, and formulas in this chapter that you need to understand.