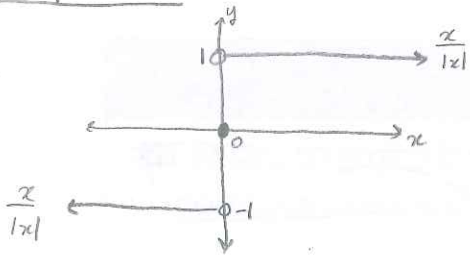
Example problems: Class 1.

1.1.17



Recall: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

so, $f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$.

Thus, $R = \{-1, 0, 1\}$.

set notation uses curly braces.

1.1.33 $f(x) = \sqrt{4 - \sqrt{x}}$. Want: 1) $x \geq 0$, so $\sqrt{x}$ will be defined

2) $4 - \sqrt{x} \geq 0$, so $\sqrt{4-\sqrt{x}}$ —— " ——, i.e., want $\sqrt{x} \leq 4$

$\Rightarrow x \leq 16$.

So, to satisfy both, need $0 \leq x \leq 16$, or $x \in [0, 16]$.

"is an element of" (a set)

1.1.37 Know: (1) $C = 2\pi r$

(2) $A = \pi r^2$

$(2) \Rightarrow r = \pm\sqrt{\frac{A}{\pi}}$, and plug this into (1): $C = \pm 2\pi\sqrt{\frac{A}{\pi}}$

"implies"

$= \pm 2\sqrt{A\pi}$.

But a circumference is a physical quantity and must be non-neg., so take only the positive square root: $C = 2\sqrt{A\pi}$.

units: length², so $[C] = $ length ✓

1.1.42 $P(x) = $ (total # of wells after adding more)(produc'n of each well after adding more)

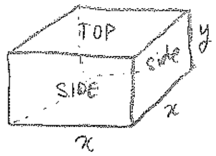
1st qty: $20 + x$ (had 20 initially, added x)

2nd qty: $\frac{4000}{20} - 5x = 200 - 5x$ (init. produc'n per well was $\frac{4000}{20}$, and it dec. by 5 barrels for each new well added)

Thus, $P(x) = \underbrace{(\underbrace{20}_{\text{wells}} + \underbrace{x}_{\text{wells}})}_{\text{wells}} \underbrace{(\underbrace{200}_{\frac{\text{barrels}}{\text{well}}} - \underbrace{5}_{\frac{\text{barrels}}{(\text{well})^2}}\underbrace{x}_{\text{well}})}_{\frac{\text{barrels}}{\text{well}}}$, so units of P are barrels ✓.

$= -5x^2 + 100x + 4000$.

1.1.46



$$A = 2(\text{Area of top}) + 4(\text{Area of side}) = 600 \text{ cm}^2$$
$$= 2x^2 + 4xy = 600 \quad (1)$$
$$V = (\text{Area of top})(\text{height})$$
$$= x^2 y. \quad (2)$$

Solve (1) for y: $y = \dfrac{600 - 2x^2}{4x} = \dfrac{300 - x^2}{2x}$

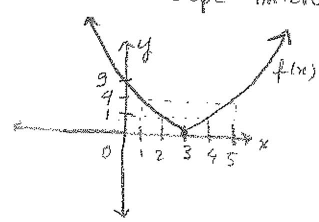
Plug into (2):

$$V = x^2\left(\frac{300 - x^2}{2x}\right) = \frac{x(300 - x^2)}{2} = -\tfrac{1}{2}x^3 + 150x.$$

1.2.5 slope $= \dfrac{3-(-3)}{5-2} = \dfrac{6}{4} = \dfrac{3}{2}$, and one pt. on the line is $(2,-3)$, so

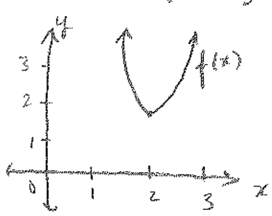
in point-slope form: $y-(-3) = \frac{3}{2}(x-2) \Rightarrow \underbrace{y = \frac{3}{2}x - 6}_{\text{slope-intercept form}}$.

1.2.17 $y = x^2 - 6x + 9 = (x-3)^2$. So $h = 3$, $k = 0$, $a = 1$ $\Rightarrow$

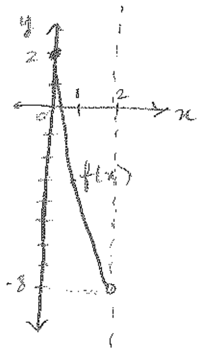


1.2.20 $2y = x^2 - 4x + 8 \overset{\text{completing the square}}{=} (x-2)^2 + 4 \Rightarrow y - 2 = 2(x-2)^2$. So $\left.\begin{matrix} h = 2 \\ k = 2 \end{matrix}\right\}$ vertex at $(2,2)$

$a = 2$ (so, skinnier parabola)

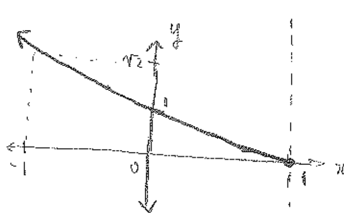


1.2.28. $f(x) = 2 - 5x, \quad 0 \le x < 2$:

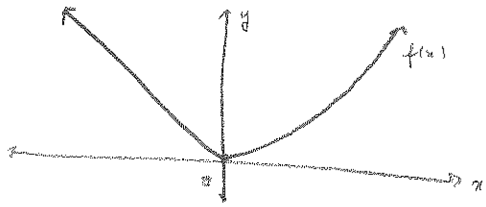


1.2.43. $f(x) = \sqrt{1-x}$.

Want $1 - x \geq 0 \Rightarrow x \leq 1$



1.2.50 $f(x) = \begin{cases} |x|, & x<0 \\ x^2, & x \geq 0 \end{cases}$. Recall: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x<0 \end{cases} \Rightarrow f(x) = \begin{cases} -x, & x<0 \\ x^2, & x \geq 0 \end{cases}$, so the plot:



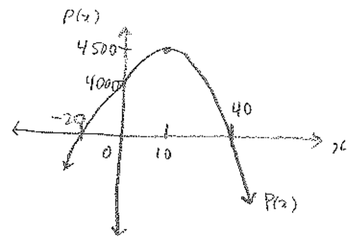
1.2.68 Recall, we had $P(x) = (20+x)(200-5x) = -5x^2 + 100x + 4000$.

So $-\frac{y}{5} = x^2 - 20x - 800 = (x-10)^2 - 900 \Rightarrow y - 5(900) = -5(x-10)^2$

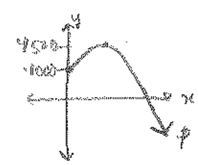
$\Rightarrow y - 4500 = -5(x-10)^2$.

Thus, $(h,k) = (10, 4500)$ and $a = -5$.

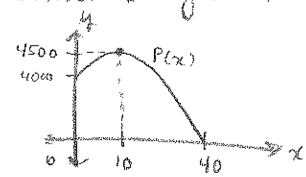
The plot of P:



But $x<0$ makes no sense, so restrict D to $[0, +\infty)$:



Also, $P<0$ makes no sense physically, so restrict D again: $D = [0, 40]$ to obtain the final graph:



The vertex — that is, the <u>maximum</u> of $P(x)$ is at $(10, 4500)$ — so, the max. daily produc'n occurs when 10 new wells are drilled, and is 4500 barrels/day.

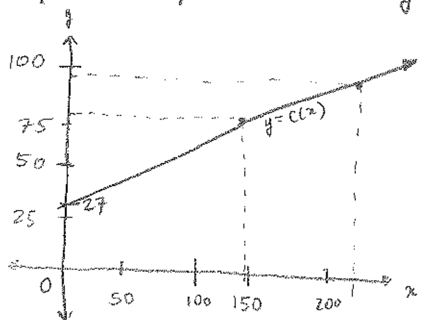
1.2.78. (a) Want a linear function $C(x)$ with the two points $(207, 99.45)$ and $(149, 79.15)$ on the line $y = C(x)$. Use point-slope form:

$$m = \frac{99.45 - 79.15}{207 - 149} = \frac{20.30}{58}, \text{ so eq'n of line is } y - 79.15 = \frac{20.3}{58}(x - 149),$$

or in slope-intercept form, $y = \frac{20.3}{58}x + 79.15 - \frac{149(20.3)}{58}$

$= 0.35x + 27.$

(b)



Intercept = fixed cost = \$27

Slope = marginal cost = 35 ¢/mile.

1.3.4 $(f \cdot g)(x) = \sqrt{(x+1)(5-x)}$; domain is where $(x+1)(5-x) \geq 0$, i.e., when $x \in [-1, 5]$.

$(f+g)(x) = \sqrt{x+1} + \sqrt{5-x}$; — " — $x+1 \geq 0$ ($\Rightarrow x \geq -1$) and $5-x \geq 0$ ($\Rightarrow x \leq 5$), — " — $x \in [-1, 5]$

$\left(\frac{f}{g}\right)(x) = \sqrt{\frac{x+1}{5-x}}$; — " — $5 - x \neq 0$ (i.e., when $x \neq 5$) and when $\frac{x+1}{5-x} \geq 0$;

both are satisfied when $x \in [-1, 5)$.

1.3.7 It's a cubic fn. w/ positive leading coefficient (so, either 1.3.26 or 1.3.30). To easily exclude 1.3.26, observe that $f(0) = 1$, but $(0,1)$ is not on 1.3.26. So 1.3.30.

1.3.8 A cubic w/ negative leading coeff. — could only be 1.3.28.

1.3.9 A quartic w/ pos. leading coeff — — " — 1.3.31.

1.3.10 A quintic w/ $f(0) = -1$. Could only be 1.3.29

1.3.11 A quartic w/ negative leading coeff — could only be 1.3.27.

1.3.12 A quintic w/ $f(0) = 0$. Could only be 1.3.26.

1.3.13 Asymptotes at $x = -1$ and $x = 2$. That's 1.3.34.

1.3.14 Asymptotes at $x = \pm 3$. That's 1.3.32.

1.3.15 No vertical asymptotes, but $f(x) \xrightarrow[x \to \pm\infty]{} 0$. That's 1.3.33.

1.3.16 Asymptote at $x = 1$. That's 1.3.35.

1.4.1. $f(0)=0$, $f(x)\xrightarrow[x\to-\infty]{}-1$. That's 1.4.29.

1.4.2 $f(0)=0$, $f(x)\xrightarrow[x\to\infty]{}2$. That's 1.4.33.

1.4.3 Looks like $\cos(x)$ shifted up by 1 unit. That's 1.4.27.

1.4.4 Looks like $\sin(x)$ with amplitude multiplied by 2, flipped over the x-axis, and shifted up by 2 units. That's 1.4.32.

1.4.5 Looks like $\cos(x)$ w/ amplitude multiplied by 2, shifted 1 unit up. That's 1.4.35

1.4.6 Looks like 1.4.4, but w/ amplitude unchanged from $\sin(x)$. That's 1.4.28.

1.4.7 $f(0)=0$, $f(x)\xrightarrow[x\to\infty]{}0$, not oscillatory. That's 1.4.31.

1.4.8 Undefined at $x=0$ (vertical asymptote), and $f(x)\xrightarrow[x\to\infty]{}0$. That's 1.4.36

1.4.9 Defined everywhere, oscillatory, $f(0)=2$, $f(x)\xrightarrow[x\to\pm\infty]{}0$. That's 1.4.34

1.4.10 Oscillatory, $f(0)=0$, $f(x)\xrightarrow[x\to+\infty]{}0$ and $f(x)\xrightarrow[x\to-\infty]{}+\infty$. That's 1.4.30.

1.4.11. $f(g(x)) = 1-(2x+3)^2$; $g(f(x)) = 2(1-x^2)+3$.

1.4.16. $f(g(x)) = \sqrt{\cos x}$, $g(f(x)) = \cos(\sqrt{x})$

1.4.22. $f(x) = x^3$, $g(x) = 4-x$.

1.4.29. $f(x) = x^{-1/2}$, $g(x) = x+10$.

1.4.42. $A(t) = 5000\cdot 1.08^t$. Ask: For which t does $A(t)=15{,}000$? Look at the graphs of $A(t)=5000\cdot 1.08^t$ and $f(t)=15{,}000$ on Wolfram Alpha — the t-value of their intersection point is the tripling time.

You can check your work algebraically: $5000\cdot 1.08^t = 15{,}000 \iff$
$1.08^t = \underbrace{\frac{15{,}000}{5{,}000}}= 3 \iff t = \log_{1.08}(3) \approx 14.2749$ years.

NOTE! The "tripling time" is <u>independent</u> of the initial investment amount. So if you deposited \$5 or \$5M, it doesn't matter — still takes about 14 years and 3 months to triple.