Class 2: Example Problems.

2.1.2 : $f(x) = x$ is of the form $f(x) = ax^2 + bx + c$, where $a = c = 0$, $b = 1$. The slope predictor is $\boxed{m(x) = 2ax + b}$, so for our case, plug in $a = c = 0$, $b = 1$ to obtain $m(x) = 1$. The tan. line intersects $(2,2)$, and has slope $m(2) = 1$, so in point-slope form, it is

$(y-2) = 1(x-2)$, or $y = x$.

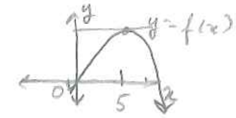
2.1.8 : $f(x) = 5 - 3x - x^2 \Rightarrow m(x) = 2(-1)x + (-3) = -2x - 3$, $m(2) = -7$,

and $f(2) = -5$, so $(y+5) = -7(x-2) \Rightarrow y = -7x + 9$.

$(2,-5)$

2.1.16 : $f(x) = 10x - x^2 \Rightarrow m(x) = -2x + 10$, which is zero when $x = 5$.

The tan. line to f is thus horizontal when $x = 5$.



2.1.24 : $f(x) = 100\left(1 - \frac{x}{10}\right)^2 = 100 - 20x + x^2 \Rightarrow m(x) = 2x - 20$, which is zero when $x = 10$. The tan. line is horizontal at $x = 10$.

2.2.2 : $\lim_{x \to -2} x^3 - 3x^2 + 5$. Root law $\Rightarrow \underline{\lim_{x \to -2} x^3 = (-2)^3 = -8}$ and $\underline{\lim_{x \to -2} x^2 = (-2)^2 = 4}$.

constant law $\Rightarrow \underline{\lim_{x \to -2} 5 = 5}$ and $\lim_{x \to -2} 3 = 3$. Product law with previous results $\Rightarrow \underline{\lim_{x \to -2} 3x^2 = 3(4) = 12}$. Sum law with underlined results implies $\lim_{x \to -2} x^3 - 3x^2 + 5 = -8 - 12 + 5 = -15$.

2.2.4 : $\lim_{x \to -2} (x^3 - 3x + 3)(x^2 + 2x + 5) = (-8 - 3(-2) + 3)(4 - 4 + 5) = 1(5) = 5$.

substitution works on polynomials

Class 2, ex. cont'd.

2.2.6: $\lim\limits_{t\to -2} \dfrac{t+2}{t^2+4} \overset{\checkmark}{=} \dfrac{-2+2}{(-2)^2+4} = \dfrac{0}{8} = 0.$ Used: quotient, root, sum, and constant laws

2.2.10: $\lim\limits_{t\to 4} \sqrt{27-\sqrt{t}} = \sqrt{27-\sqrt{4}} = \sqrt{25} = 5.$ Used: composi'n, root, sum, constant laws.

2.2.20: $\lim\limits_{t\to 3} \dfrac{t^2-9}{t-3} = \lim\limits_{t\to 3} \dfrac{\cancel{(t-3)}(t+3)}{\cancel{t-3}} = \lim\limits_{t\to 3} t+3 = 6.$ (POLYNOMIAL)

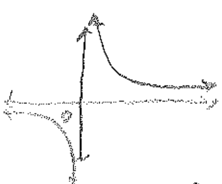
2.2.26: $\lim\limits_{t\to 3} \dfrac{t^3-9t}{t^2-9} = \lim\limits_{t\to 3} \dfrac{t\cancel{(t^2-9)}}{\cancel{t^2-9}} = \lim\limits_{t\to 3} t = 3.$

2.2.29: $\lim\limits_{x\to 3} \dfrac{\frac{1}{x}-\frac{1}{3}}{x-3} = \lim\limits_{x\to 3} \left(\dfrac{1}{x}-\dfrac{1}{3}\right)\dfrac{1}{x-3} = \lim\limits_{x\to 3} \dfrac{\cancel{3-x}}{3x\cancel{(x-3)}} =$

$= \lim\limits_{x\to 3} \dfrac{-1}{3x} = \dfrac{-1}{9}. \qquad \overset{\checkmark}{=} \dfrac{\lim\limits_{x\to 3} -1}{\lim\limits_{x\to 3} 3x}$

2.2.32 $\lim\limits_{x\to 9} \dfrac{3-\sqrt{x}}{9-x} = \lim\limits_{x\to 9} \dfrac{\cancel{3-\sqrt{x}}}{\cancel{(3-\sqrt{x})}(3+\sqrt{x})} = \lim\limits_{x\to 9} \dfrac{1}{3+\sqrt{x}} = \dfrac{1}{6}.$

2.2.38 $f(x) = \dfrac{1}{x}$. $\quad m(x) = \lim\limits_{\Delta x\to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x} = \lim\limits_{\Delta x\to 0} \left(\dfrac{1}{x+\Delta x} - \dfrac{1}{x}\right)\left(\dfrac{1}{\Delta x}\right) =$



$= \lim\limits_{\Delta x\to 0} \dfrac{\cancel{x}-1(\cancel{x}+\cancel{\Delta x})}{x\cancel{\Delta x}(x+\Delta x)} = \lim\limits_{\Delta x\to 0} \dfrac{-1}{x(x+\cancel{\Delta x})} = \dfrac{-1}{x^2}.$

$(2, \tfrac{1}{2})$

So $m(2) = \dfrac{-1}{2^2} = -\dfrac{1}{4}$, and $f(2) = \dfrac{1}{2}$, so tan. line to f at $x=2$

is $\quad y-\frac{1}{2} = -\frac{1}{4}(x-2) \quad$ or $\quad y = -\frac{1}{4}x + 1.$

2.3.2 $\lim\limits_{\theta\to 0} \dfrac{\sin^2\theta}{\theta^2} \overset{\checkmark}{=} \left(\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta}\right)\underbrace{\left(\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta}\right)} = 1(1) = 1.$

Gk. "theta" $\qquad$ if both exist (prod. rule) $\qquad$ RULE: $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1.$

2.3.4 $\lim\limits_{\theta\to 0} \dfrac{\tan\theta}{\theta} = \lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta\cos\theta} = \underbrace{\left(\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta}\right)}_{1}\underbrace{\left(\lim\limits_{\theta\to 0} \dfrac{1}{\cos\theta}\right)}_{1} = 1(1) = 1.$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Class 2: Ex. probs, ct'd.

2.3.22 $\lim\limits_{x\to 0} \dfrac{x - \tan x}{\sin x} \underset{\substack{\uparrow \\ \text{if all lims} \\ \text{on right exist}}}{=} \lim\limits_{x\to 0} \dfrac{x}{\sin x} - \lim\limits_{x\to 0} \dfrac{\tan x}{\sin x} = \dfrac{1}{\lim\limits_{x\to 0} \frac{\sin x}{x}} - \lim\limits_{x\to 0} \dfrac{1}{\cos x}$

$\dfrac{\tan x}{\sin x} = \dfrac{1 \cdot \cancel{\sin x}}{\cos x \cancel{\sin x}}$

$= \dfrac{1}{1} - 1 = 0.$

2.3.26 $g(x) = x^2 \sin \frac{1}{x^2}$. Note that $-x^2 \le x^2 \sin \frac{1}{x^2}$ (because sin is bounded below by -1)

and $x^2 \sin \frac{1}{x^2} \le x^2$ (because sin is bd. above by 1).

So take $f(x) = -x^2$, $g(x) = x^2 \sin \frac{1}{x^2}$, $h(x) = x^2$.

We have $f(x) \le g(x) \le h(x)$ in all $\underbrace{\text{deleted nbds.}}_{\text{means nbds. that don't include } x=0 \text{ itself}}$ of $x = 0$,

and we have $\lim\limits_{x\to 0} \underbrace{-x^2}_{f(x)} = 0 = \lim\limits_{x\to 0} \underbrace{x^2}_{h(x)}$.

Therefore, by the squeeze theorem, have $\lim\limits_{x\to 0} \overbrace{x^2 \sin \frac{1}{x^2}}^{g(x)} = 0$ as well.