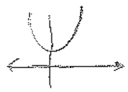
Lect. 3: Problems.

2.4.3 $g(x) = \dfrac{2x-1}{4x^2+1}$. First consider $f_1(x) = 2x-1$ and $f_2(x) = 4x^2+1$.

Each is a polynomial, so each is cts. for all $x \in \mathbb{R}$.



Also, $f_2(x) \neq 0$ anywhere in $\mathbb{R}$, so $f(x) = \dfrac{f_1(x)}{f_2(x)}$ is cts. for all $x \in \mathbb{R}$ by the quotient rule.

2.4.39 $f(x) = \dfrac{x-2}{x^2-4} = \dfrac{\cancel{x-2}}{(x+2)\cancel{(x-2)}}$. Is not defined at $x=-2$ and at $x=+2$;

the discontinuity at $x=+2$ is removable (factor can be cancelled), but at $x=-2$ is not removable (it is an <u>infinite discontinuity</u>).

2.4.50 $f(x) = \begin{cases} 2x+c, & x \le 3 \\ 2c-x, & x > 3. \end{cases}$ Want $2x+c\Big|_{x=3} = 2c-x\Big|_{x=3}$

$\iff 2(3)+c = 2c-3$

$\iff 6+c = 2c-3$

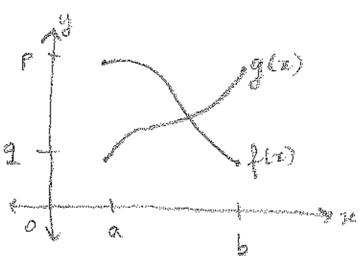
$\iff 9 = c$.

2.4.54. $f(x) = x^3+x+1$. Note that as it is a polynomial, $f(x)$ is cts. for all $x \in \mathbb{R}$, and in particular, on the closed interval $[-1,0]$.

Note $f(-1) = (-1)^3+(-1)+1 = -1$, and $f(0) = 0^3+0+1 = 1$, and $-1<0<1$.

By the IVT, $\exists c \in [-1,0]$ s.t. $f(c) = 0$.

2.4.63.



Note, $h(x) = f(x) - g(x)$ is cts. on $[a,b]$, as f and g were (difference rule for continuity), and

$h(a) = f(a) - g(a) = p - q$, and

$h(b) = f(b) - g(b) = q - p$. Note that 0 lies between $p-q$ and $q-p$, so for some $x \in [a,b]$, $h(x) = 0$, and thus $f(x) = g(x)$.

2.4.64. Is just 2.4.63 with $a=1$, $b=2$, and f and g both <u>cts.</u> fns. describing position of your car w.r.t. time. (Continuity is key).

3.1.2 $g(t) = \underbrace{100}_{c} - \underbrace{16}_{a} t^2 \underbrace{}_{b=0} \Rightarrow g'(t) = 2(-16)t + 0 = -32t$.

3.1.7. $z = 5u^2 - 3u \Rightarrow \frac{dy}{du} = 2(5)u - 3 = 10u - 3$.

3.1.12 $f(x) = 2 - 3x \Rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{2 - 3(x+\Delta x) - (2-3x)}{\Delta x}$

$$= \lim_{\Delta x \to 0} \frac{2 - 3x - 3\Delta x - 2 + 3x}{\Delta x} = \lim_{\Delta x \to 0} \frac{-3\Delta x}{\Delta x} = \lim_{\Delta x \to 0} -3 = -3.$$

3.1.16 $f(x) = \frac{1}{3-x} \Rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{\frac{1}{3-x-\Delta x} - \frac{1}{3-x}}{\Delta x} = \lim_{\Delta x \to 0} \frac{3-x-(3-x-\Delta x)}{\Delta x(3-x)(3-x-\Delta x)} = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta x(3-x)(3-x-\Delta x)}$

$= \lim_{\Delta x \to 0} \frac{1}{(3-x)(3-x-\Delta x)}$. Note that $(3-x)(3-x-\Delta x)$ is a polynomial in Δx that is nonzero when $\Delta x = 0$, It is thus cts. at $\Delta x = 0$, so the limit can be evaluated by substitution, i.e.,

$$f'(x) = \lim_{\Delta x \to 0} \frac{1}{(3-x)(3-x-\Delta x)} = \frac{1}{(3-x)^2}.$$

3.1.22 $x = -16t^2 + 160t + 25 \Rightarrow v(t) = -16(2)t + 160 = 160 - 32t$.

So $v(t) = 0$ when $160 - 32t = 0 \Leftrightarrow t = \frac{160}{32} = 5$ sec., so the location when the velocity is zero is $x(5) = -16(5)^2 + 160(5) + 25 = 425$ (ft., meters, etc.)

3.1.30. Should have $f'(0) = 0$, and no other zeros; that's fig. (a)

3.1.31. — " — $f'(1) = 0$, and no other zeros; that's fig. (c)

3.1.32 — " — $f'(-1) = f'(1) = 0$, — " — ; that's fig (b)

3.1.33 — " — $f(-1) = f(0) = f(1) = 0$, — " — ; — " — (f)

3.1.34. — " — $f(-4) = f(0) = f(4) = 0$, — " — ; — " — (a)

3.1.35 — " — no zeros – that's fig. (d).

3.1.50. $y(t) = -16t^2 + 96t + 112$. (a) This is asking for the position attained when the velocity is zero. Note, $v(t) = -16(2)t + 96 = 96 - 32t$, so $v(t) = 0$ when $96 - 32t = 0$, i.e., when $t = 3$ sec. Now, $y(3) = -16(3)^2 + 96(3) + 112 = 256$ ft.

(b) This is asking for the speed when the position is zero. Note, $y(t) = 0$ when $t = \frac{-96 \pm \sqrt{96^2 - 4(112)(-16)}}{2(-16)} = \frac{-96 \pm 128}{-32} = -1$ or 7. We're interested only in $t = 7$, so we compute $v(7) = 96 - 32(7) = -128$ ft/sec. The speed is thus $|-128 \text{ ft/sec}| = 128$ ft/sec.

3.2.1 $f(x) = 3x^2 - x + 5$

$$\frac{df}{dx} = \frac{d}{dx}[3x^2] + \frac{d}{dx}[-x] + \frac{d}{dx}[5]$$ Addition

$$= 3\frac{d}{dx}[x^2] - \frac{d}{dx}[x^1] + \frac{d}{dx}[5]$$ Const. mult. rule

(lin. comb. rule)

$$= 3[2x^1] - [1] + 0$$ Power rule & const. rule

$$= 6x - 1.$$

3.2.6 $g(t) = (4t-7)^2 \Rightarrow g'(t) = (4t-7)\frac{d}{dt}[4t-7] + (4t-7)\frac{d}{dt}[4t-7]$ Prod. rule

$$= (4t-7)(4) + (4t-7)(4)$$ Power rule & const. rule & sum rule

$$= 8(4t-7) = 32t - 56.$$

3.2.18. $f(x) = \frac{x^2-4}{x^2+4} \Rightarrow f'(x) = \frac{(x^2+4)\frac{d}{dx}[x^2-4] - (x^2-4)\frac{d}{dx}[x^2+4]}{(x^2+4)^2}$ Quotient rule

$$= \frac{(x^2+4)(2x) - (x^2-4)(2x)}{(x^2+4)^2}$$ Power rule & const. rule & sum rule

$$= \frac{2x(x^2+4-x^2+4)}{(x^2+4)^2} = \frac{2x(8)}{(x^2+4)^2} = \frac{16x}{(x^2+4)^2}.$$

3.2.27. $h(x) = x^3 - 6x^5 + \frac{3}{2}x^{-4} + 12$

$$\Rightarrow h'(x) = \frac{d}{dx}[x^3] - 6\frac{d}{dx}[x^5] + \frac{3}{2}\frac{d}{dx}[x^{-4}] + \frac{d}{dx}[12]$$ Lin. comb. rule

$$= 3x^2 - 6(5x^4) + \frac{3}{2}(-4x^{-5}) + 0$$ Power rule (n.b.— it still applies when $n < 0$)

$$= 3x^2 - 30x^4 - 6x^{-5}.$$

3.2.28. $x(t) = \frac{3}{t} - \frac{4}{t^2} - 5 = 3t^{-1} - 4t^{-2} - 5$

$$\Rightarrow x'(t) = 3\frac{d}{dt}[t^{-1}] - 4\frac{d}{dt}[t^{-2}] - \frac{d}{dt}[5]$$ Lin. comb. rule

$$= 3(-t^{-2}) - 4(-2t^{-3}) - 0$$ Power rule

$$= -\frac{3}{t^2} + \frac{8}{t^3}.$$

3.2.32 $f(z) = \dfrac{1}{z(z^2+2z+2)} = \dfrac{1}{z} \cdot \dfrac{1}{z^2+2z+2}$

$$\Rightarrow \quad f'(z) = \frac{1}{z} \frac{d}{dz}\left[\frac{1}{z^2+2z+2}\right] + \frac{1}{z^2+2z+2} \frac{d}{dz}\left[\frac{1}{z}\right] \qquad \text{Prod. rule}$$

$$= \frac{1}{z} \cdot \frac{(z^2+2z+2)\frac{d}{dz}[1] - 1\frac{d}{dz}[z^2+2z+2]}{(z^2+2z+2)^2} + \frac{1}{z^2+2z+2} \frac{d}{dz}\left[\frac{1}{z}\right] \qquad \text{Quot. rule}$$

$$= \frac{-1\left(\frac{d}{dz}[z^2]+2\frac{d}{dz}[z]+2\frac{d}{dz}[1]\right)}{z(z^2+2z+2)^2} + \frac{1}{z^2+2z+2} \frac{d}{dz}\left[\frac{1}{z}\right] \qquad \text{Const. rule \& lin. comb. rule}$$

$$= \frac{-(2z+2)}{z(z^2+2z+2)^2} + \frac{1}{z^2+2z+2}\left(-\frac{1}{z^2}\right) \qquad \text{Power rule}$$

$$= \frac{-z(2z+2)-1}{z^2(z^2+2z+2)^2} = \frac{-2z^2-2z-1}{z^2(z^2+2z+2)^2}$$

3.2.44 $y = 2x - \frac{1}{x}$, $P(0.5, -1)$. Then $\frac{dy}{dx} = 2\frac{d}{dx}[x] - \frac{d}{dx}[x^{-1}]$

$$= 2 - (-x^{-2}) = 2 + \frac{1}{x^2},$$

and so $\left.\frac{dy}{dx}\right|_{x=0.5} = 2 + \frac{1}{(\frac{1}{2})^2} = 2 + 2^2 = 6.$

Thus, the eq'n of the tangent line is $y+1 = 6(x-0.5)$,

or $6x - y = 4$.

3.2.60 $y = x^5 + 2x$. Show $\nexists\, a \in \mathbb{R}$ s.t. $\left.\frac{dy}{dx}\right|_{x=a} = 0$. Well, $\frac{dy}{dx} = 5x^4 + 2$, (polynomial rule)

So we seek to show $5x^4 + 2 \neq 0$ for any x. Let $z := x^2$, so that $x = \pm\sqrt{z}$.

Then $5x^4 + 2 = 0 \Leftrightarrow 5z^2 + 2 = 0 \Leftrightarrow z = \pm\underbrace{\sqrt{-\frac{2}{5}}}$

which has no real solutions.

So x has no real solutions.

(Could have graphed to find that $y = 5x^4 + 2$ is always above the x-axis.)