Lecture 2: Example problems.

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3.3.2 $y = (2-5x)^3$. Let $g(x) = 2-5x$, $f(u) = u^3$, so $y = f(g(x))$.

Then $\underline{\frac{dy}{dx} = f'(g(x))\,g'(x)}$ by the chain rule, and we substitute the values:

$f'(u) = 3u^2 \Rightarrow f'(g(x)) = 3(2-5x)^2$.

$g'(x) = -5$.

So $\frac{dy}{dx} = \underbrace{\left[3(2-5x)^2\right]}\underbrace{\left[-5\right]} = -15(2-5x)^2$

3.3.3 $y = \frac{1}{(2x+1)^3} = (2x+1)^{-3}$. Let $g(x) = 2x+1$, $f(u) = u^{-3}$, so that

$f'(u) = -3u^{-4} \Rightarrow f'(g(x)) = -3(2x+1)^{-4} = \frac{-3}{(2x+1)^4}$

$g'(x) = 2$.

So, $\frac{dy}{dx} = \underbrace{f'(g(x))}\underbrace{g'(x)} = \left[\frac{-3}{(2x+1)^4}\right]\left[2\right] = \frac{-6}{(2x+1)^4}$.

3.3.16 $y = u^5$, $\boxed{u = \frac{1}{3x-2}} \Rightarrow \frac{dy}{du} = 5u^4$, $\frac{du}{dx} = \frac{(3x-2)(0) - 1(3)}{(3x-2)^2} = \frac{-3}{(3x-2)^2}$.

$\frac{dy}{dx} = \frac{dy}{du}\,\frac{du}{dx} = \left[5u^4\right]\left[\frac{-3}{(3x-2)^2}\right]$

$= \left[\frac{5}{(3x-2)^4}\right]\left[\frac{-3}{(3x-2)^2}\right] = \frac{-15}{(3x-2)^6}$.

3.3.22. $f(x) = \frac{1}{2+5x^3} = (2+5x^3)^{-1}$. Let $y = u^{-1}$ and $u = 2+5x^3$.

Then $\frac{dy}{du} = -u^{-2} = -(2+5x^3)^{-2} = \frac{-1}{(2+5x^3)^2}$

and $\frac{du}{dx} = 5\cdot 3x^2 = 15x^2$.

So $f'(x) = \underline{\frac{dy}{du}\,\frac{du}{dx}} = \left[\frac{-1}{(2+5x^3)^2}\right]\left[15x^2\right] = \frac{-15x^2}{(2+5x^3)^2}$.

Example probs, ct'd.

3.3.45 $f(x) = \sin(x^3)$. Take $y = \sin(u)$, $u = x^3$.

Then $\frac{dy}{du} = \cos(u) = \cos(x^3)$, and $\frac{du}{dx} = \underline{3x^2}$.

Thus, $f'(x) = \frac{dy}{du}\frac{du}{dx} = 3x^2\cos(x^3)$.

3.3.46 $f(x) = (\sin x)^3$. Take $y = u^3$, $u = \sin x$.

Then $\frac{dy}{du} = 3u^2 = 3(\sin x)^2$, and $\frac{du}{dx} = \cos x$.

Thus, $f'(x) = \frac{dy}{du}\frac{du}{dx} = 3\cos x\,(\sin x)^2$.

3.7.10 $g(t) = (2-\cos^2 t)^3$. Let $u(t) = 2-\cos^2 t$, let $y(u) = u^3$. Then

$g(t) = y(u(t)) \Rightarrow g'(t) = \frac{dy}{du}\frac{du}{dt} = 3u^2 \frac{d}{dt}\left[2-\underbrace{\cos^2 t}\right] = 3u^2(-2\sin t\cos t) =$

Aside. Let $w(t) = \cos t$
$z(w) = w^2$.

Then $z'(w(t)) = 2w(t) = 2\cos t$; $\Rightarrow$
$w'(t) = -\sin t$

$\Rightarrow \frac{d}{dt}[\cos^2 t] = -2\sin t\cos t$

$= 3(2-\cos^2 t)^2(-2\sin t\cos t)$

$= -6(2-\cos^2 t)\sin t\cos t$.

3.7.22 $y = \frac{\cos(2x)}{x} \Rightarrow \frac{dy}{dx} = \frac{x\frac{d}{dx}[\cos(2x)] - \cos(2x)\frac{d}{dx}(x)}{x^2}$

$= \frac{x\frac{d}{dx}(\cos 2x) - \cos(2x)}{x^2}$

$= \frac{-2x\sin(2x) - \cos(2x)}{x^2}$.

Aside. Let $u(x) = 2x$
$y(u) = \cos u$

Then $y'(u(x)) = -\sin u(x)$
$= -\sin(2x)$

; $u'(x) = 2$

so $\frac{d}{dx}[\cos(2x)] = -2\sin(2x)$

3.7.50. $x = \cot\left(\frac{1}{\sqrt{t}}\right) \Rightarrow \frac{dx}{dt} = -\csc^2\left(\frac{1}{\sqrt{t}}\right)\underbrace{\frac{d}{dt}\left(\frac{1}{\sqrt{t}}\right)}$

Aside: $\frac{d}{dt}\left(\frac{1}{\sqrt{t}}\right) = \frac{d}{dt}\left(t^{-1/2}\right) = -\frac{1}{2}t^{-3/2}$
$= -\frac{1}{2t^{3/2}}$

$= \frac{\csc^2(1/\sqrt{t})}{2t^{3/2}}$.

Used Rule (10) from text

Problems, ct'd.

3.8.2 $f(x) = e^{3x-1} \Rightarrow f'(x) = \frac{d}{dx}(3x-1)\, e^{3x-1} = 3e^{3x-1}$

3.8.6 $f(x) = x^2 e^{(x^3)} \Rightarrow f'(x) = 2x e^{(x^3)} + x^2 \frac{d}{dx}(x^3)\, e^{(x^3)}$

$$= 2xe^{(x^3)} + x^2(3x^2)\, e^{(x^3)}$$

$$= 2xe^{(x^3)} + 3x^4 e^{(x^3)} = (2+3x^3)\, x e^{(x^3)}$$

3.8.10 $f(t) = \sqrt{e^t - e^{-t}} \Rightarrow f'(t) = \frac{1}{2}(e^t - e^{-t})^{-1/2} \frac{d}{dt}(e^t - e^{-t})$ chain rule

$$= \frac{e^t - (-1)e^{-t}}{2\sqrt{e^t - e^{-t}}} = \frac{e^t + e^{-t}}{2\sqrt{e^t - e^{-t}}}$$

3.8.12 $g(x) = x e^{\sin x} \Rightarrow g'(x) = x \frac{d}{dx}\left[e^{\sin(x)}\right] + (1) e^{\sin x}$

$$= x \frac{d}{dx}[\cos x]\, e^{\sin x} + e^{\sin x}$$

$$= -x \sin x\, e^{\sin x} + e^{\sin x} = e^{\sin x}(1 - x\sin x)$$

3.8.20 $f(x) = \cos(e^x + e^{-x}) \Rightarrow f'(x) = -\sin(e^x + e^{-x})$