Calculus I
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
May 18, 2014

Week 1: Reading, Practice Problems, and Homework Exercises

How This Works

Each week, before Monday morning, the week's reading and homework problems will be posted to the course's myWPI page (`my.wpi.edu`). In mathematics, reading without working through problems is useless. So a number of interesting exercises are also assigned. Solving them is not usually hard, but writing them up neatly and meeting a deadline is less pleasant. So, while you are expected to look at all of the practice problems each week, your homework will only consist of only a few problems per week. These are intended to be more thought-provoking, and you are expected to complete these problems with great care and to present them in a professional way. You are encouraged to type your solutions using LaTeX, or to handwrite them very neatly and scan them at a sufficiently high resolution as to be legible. If your work is not presentable or if it is illegible, you will not receive credit for it.

These submitted solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

Reading

Please read Sections 1–4 of Chapter 1 in time for Monday's lecture, and please read Section 1.5 and Sections 2.1 and 2.2 before Wednesday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

True/False Study Guides

Please find at the end of each section, before the problems are given, the True/False Study Guide for that section. You should read through these true/false items to check your understanding of the section, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be relatively simple problems just for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 1.1, Problems 3–27 odd (easy/review problems); 36–40, 54 and 55, 57–61 odd
- Section 1.2, Problems 1–21 odd; 27–39 odd, 51 and 52, 57 and 69, 67, 77
- Section 1.3, Problems 1–25 odd; 31–37
- Section 1.4, Problems 1–10, 17–23 odd, 29 (for 21, 23, and 29, assume $k \neq 1$), 31–39 odd
- Section 2.1, Problems 1, 3, 5, 13, 15, 19, 27, 30, 31, 37–47 odd
- Section 2.2, Problems 1–17 odd, 19, 27, 43, 53, 59

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ______________________

WPI Username: ______________________

Week 1: Homework Problems

Due date: Saturday, 24 May 2014, 12:00 a.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) No late assignments will be accepted for credit.

Problem 1. **(a)** An oil field containing 50 wells has been producing 10,000 barrels of oil daily. For each new well that is drilled, the daily production of each well decreases by 2 barrels per day. Write the total daily production P of the oil field as a function $P = f(x)$ of the number x of new wells drilled, and clearly explain your reasoning.

(b) What are the independent and dependent variables of f, and what are their units?

(c) State the domain of f as a mathematical function, and state the domain of f that is practically relevant to the scenario being modelled. Explain why these differ.

(d) Use the technique called "Completing the Square"[2] to get f into a form whose graph you can draw using the technique described on Page 18 of the text. Draw the graph of f, including all labels, arrows, and axis markings.

(e) Determine the maximum daily production $P_{\max}$ of the oil field, and $x_{\max}$, the number of additional wells that should be drilled in order to produce that maximum daily output. Show these values on the graph you drew for part (c).

(f) State the range of f, using both set notation and interval notation. Remember that the range of output values of a function generally depends on the domain to which that function's input values are restricted. Do the two different domains you found in part (c) also correspond to different ranges of f?

(g) When considering the function f that is practically relevant to the scenario being modelled, state how you would place a further restriction the domain in order to keep the range values practically sensible as well.

Problem 2. Currently, the average annual rate of interest on Vanguard's VBLTX (long-term bond index fund) is 7.67%.[3]

(a) Suppose that you invest \$5,500 at one time this year in VBLTX, and that you stop contributing to this account afterward. If interest is compounded annually, and if the annual rate of interest stays fixed at 7.67% per year, how much money will you have in that account forty years from now? (See Example 8 on Page 39.)

(b) Suppose that you wait ten years before investing your \$5,500. How much money will you have forty years from now in that case? (That is, how much will you have thirty years after you start investing?)

(c) Suppose that beginning now, you invest \$5,500 per year in VBLTX for the next fifteen years (you deposit a total of fifteen times). How much money will you deposit over that period of time? (Do not compute interest yet.) If the interest rate remains fixed at 7.67% and interest is compounded annually, compute the interest to find out how much money will be in your account at the end of the fifteenth year.

[2] If you need a refresher on this, see `http://en.wikipedia.org/wiki/Completing_the_square#Overview`

[3] The average is taken over all the years that Vanguard has had this fund available for customers to invest in—that is, since March 1994. You can see the return rates for some of Vanguard's other mutual funds here: `https://investor.vanguard.com/mutual-funds/vanguard-mutual-funds-list`. The returns on some of their funds are lower than those on VBLTX, and others are higher.

Hint: The amount of money in your account will be the sum of fifteen terms:

$$\sum_{i=1}^{15} 5500 \cdot 1.0767^i = \underbrace{5500 \cdot 1.0767^1}_{\text{last deposit's contribution}} + 5500 \cdot 1.0767^2 + \cdots + 5500 \cdot 1.0767^{14} + \underbrace{5500 \cdot 1.0767^{15}}_{\text{first deposit's contribution}}.$$

You might find a tool like WolframAlpha useful in computing this sum; use this example as a template: `http://www.wolframalpha.com/input/?i=%5Csum_%7Bi%3D0%7D%5E%7B20%7D+43*1.06%5Ei`, and change the numbers in it to obtain the sum above (the sum is under the "Decimal Form" heading).

(d) Continuing the scenario in part (c), suppose that you never make another contribution to that account after those first fifteen years go by, but that the interest rate remains fixed at the same rate of 7.67% and compounds annually for the next thirty years after. How much will be in that account after those next thirty years have passed?

(e) Suppose that your friend never thinks very much about his retirement now, and after his mid-life crisis in fifteen years, he begins to invest \$5,500 per year in VBLTX, with the same fixed 7.67% rate of annually compounded interest, for the next thirty years after. How much money will he deposit in total? How much money will be in his account forty-five years from now, when you are both ready to retire? (Similar Hint: The total amount in his account at that time will be the sum of thirty terms—do this with WolframAlpha like you did part (c).)[4]

Problem 3. Recall the quadratic function that you found in Problem 1, part (a) to describe the production of an oil field.

(a) Use Equation (10) on Page 59, or Equation 16 on Page 71 (your choice[5]) to write the slope-predictor function m of P.

(b) For which value of x is the tangent line to the parabola P horizontal? (Set $m(x)$ from part (a) equal to zero, and solve for x.) Does this agree with the maximum that you found in Problem 1, part (c)?

(c) On the same graph that you made for Problem 1, please add the plot of the line $m(x)$. Note that when $m(x) > 0$, the slope of the line tangent to the parabola $P(x)$ is also positive—we say here that P is *increasing*, and conversely, we say that $P(x)$ is *decreasing* when $m(x) < 0$. Can you see this on the graph of P?

(d) What do you think could be a good general rule for where a local maximum or minimum value of a function $f(x)$ occurs? In your answer, please mention the slope $m(x)$ of the line tangent to f at x. (Hint: Think of a function's local maximum as a point where that function ceases to increase, and begins to decrease; conversely, think of a function's local minimum as a point where that function ceases to decrease, and begins to increase.) This answer is just supposed to be an informed guess on your part, and you will receive full credit as long as your answer makes sense—so please do not spend a lot of time looking up any general theorems or statements of this kind, unless you just cannot stop yourself. We will cover this topic in class shortly.

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[4] I hope that doing this problem makes you think about how important it is to begin investing early. The student loan interest you pay won't be compounding for the rest of your life, but the interest your earn on your savings will—and, to boot, yearly IRA contributions are currently limited by U.S. law (to \$5,500 per year for people in your instructor's age and income bracket), making it truly impossible to catch up in the future if you fall behind now. So, learn more about IRAs here: `http://money.cnn.com/retirement/guide/IRA_Basics.moneymag/index.htm`, and begin investing while you're young (:

[5] Your answer will be the same no matter which equation you choose; Example 11 on Page 72 makes it clear why this is.