Calculus I
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
May 27, 2014

Week 2: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Section 4 of Chapter 2 and Sections 1 and 2 of Chapter 3 in time for Wednesday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

True/False Study Guides

Please find at the end of each section, before the problems are given, the True/False Study Guide for that section. You should read through these true/false items to check your understanding of the section, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be relatively simple problems just for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 2.3, Problems 1–11 odd; 25–28, 29–39 odd, 45, 47; 49–53 odd; 59, 60
- Section 2.4, Problems 1–13 odd; 23–35 odd; 37–45 odd; 51; 53–57 odd
- Section 3.1, Problems 1–9 odd (easy/review of slope predictors); 15–19 odd; 21; 27; 30–35; 40
- Section 3.2, Problems 1–40 odd; 45–49 odd

[1]See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ______________________

WPI Username: ______________________

Week 2: Homework Problems

Due date: Saturday, 31 May 2014, 12:00 a.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. Recall the "epsilon-delta" definition of the limit:

> Suppose that $f(x)$ is defined in an open interval containing the point a (except possibly not at a itself). Then we say that the number L is the *limit of* $f(x)$ *as* x *approaches* a—and we write
>
> $$\lim_{x \to a} f(x) = L$$
>
> —provided that the following criterion is satisfied: Given any number $\varepsilon > 0$, there exists a corresponding number $\delta > 0$ such that
>
> $$\text{if } 0 < |x - a| < \delta, \text{ then } |f(x) - L| < \varepsilon.$$

(a) Use this definition to prove that $\lim_{x \to 3} 3x + 7 = 16$.

(b) How would you **negate** the epsilon-delta definition of the limit? That is, how would you go about proving that, for some given f, a, and L, $\lim_{x \to a} f(x) \neq L$, using epsilons and deltas?

(c) Formulate precise "epsilon-delta" definitions of the one-sided limits (that is, formulate one definition for the left-hand limit, and one for the right-hand limit).

(d) Formulate a precise "M-delta" definition of the infinite limit $\lim_{x \to a} f(x) = +\infty$. Your definition should involve the inequality $f(x) > M$.

Problem 2. **(a)** Prove, using the definition of continuity, that the function $f(x) = 6^x$ is continuous everywhere on the real line.

(b) Prove, using the Intermediate Value Theorem, that there is a positive, real solution of the equation $x^3 + 3 = 6^x$.

Problem 3. **(a)** Establish, using the definition of the derivative, that the derivative of $f(x) = \frac{c}{x}$ is $f'(x) = -\frac{c}{x^2}$, if c is a constant.

(b) The volume V (in liters) of 3 g of CO_2 at 27°C is given n terms of its pressure p (in atmospheres) by the formula

$$V = \frac{1.68}{p}.$$

What is the rate of change of V with respect to p when $p = 2$ atm?

(c) Plot $V(p)$ and $V'(p)$ on the same graph. Be sure to include all appropriate axis labels, arrows, and scale markings.