Calculus I
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
June 01, 2014

Week 3: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 3, 7, and 8 of Chapter 3 in time for Monday's lecture, and Sections 4, 5, and 6 of Chapter 3 in time for Wednesday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

True/False Study Guides

Please find at the end of each section, before the problems are given, the True/False Study Guide for that section. You should read through these true/false items to check your understanding of the section, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be relatively simple problems just for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 3.3, Problems 1–19 odd; 23, 26, 37–47 odd, 51
- Section 3.7, Problems 1–15 odd; 33–49 odd; 61–68; 74
- Section 3.8, Problems 1–21 odd; 39–43 odd; 47–53 odd; 63–65
- Section 3.4, Problems 1–13 odd; 41–49 odd; 57–62
- Section 3.5, Problems 1–21 odd; 44; 47–52
- Section 3.6, Problems 1–9 odd; 27; 35; 48, 49

[1]See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ____________________

WPI Username: ________________

Week 3: Homework Problems

Due date: Monday, 09 June 2014, 12:00 a.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. Prove, using the definition of the derivative along with a trigonometric sum-to-product formula and some pre-established trigonometric limits, that $\frac{\mathrm{d}}{\mathrm{d}x}[\cos(x)] = -\sin(x)$. For this problem, I will be strictly enforcing the rule about using complete sentences, so I do **not** want to see just a string of equalities with no context. Please consult the similar proof on Page 170 of the text for an example of the type of proof you will be expected to write for this. Borrow the phrasing of the textbook if you must, or try to improve upon it if you can, modifying the argument to fit this problem.

Problem 2. The Robbins cinquefoil (*Potentilla robbinsiana*) is a small, extremely rare, yellow-flowered alpine plant found exclusively in New Hampshire's White Mountains. It was on the list of endangered species from the 1980s until 2002, when recovery efforts became successful[2].

(a) At the time the Robbins cinquefoil was taken off the endangered species list, there were about 14,000 plants in a single group on Mount Washington. If we assume that the growth of the plant population is exponential and that the population of this group will double every six years, then a formula for the number of plants at time t, where t represents the number of years after June 2002, is

$$N(t) = 14,000 \cdot 2^{t/6}.$$

How many plants do we expect to be in this group today, exactly twelve years after June 2002?

(b) When do we expect 100,000 plants to be in this group? (Please give a year and a month.)

(c) How fast is the number of plants expected to grow now (twelve years after 2002), and how fast is it expected to grow two years from now (fourteen years after 2002)? Please give your answer in units of plants per year.[3]

[2] The information about this peculiar little plant was taken from the following site, where you can read more about its ecology: `http://www.centerforplantconservation.org/collection/cpc_viewprofile.asp?CPCNum=3609`

[3] Hint: this problem asks for the instantaneous rates of change of the function.

Problem 3. You have decided to abandon your science education and go into business selling vegetarian burritos out of a food truck in Manhattan. Suppose that your fixed costs are \$500 per day, and that each burrito costs you \$1.00 to make.

(a) Let b be the number of burritos that you sell per day, and write your costs per day as a function of b.

(b) If you charge a price of p dollars per burrito, then write your gross daily income as a function of b (supposing that p is constant), and write your net daily profit as a function of the two variables b (your net profit is the gross income, minus the total costs). Call the net daily profit function $M(b)$, and note that your expression for $M(b)$ should involve p, which for now, we are supposing is constant.

(c) Of course, the more money you charge for your burritos, the fewer people will want to buy them. Suppose that you've been winging it for the first few weeks, but using this time to record valuable sales data, and you've found that if you control for all other factors, you will sell 150 burritos if you charge \$7.00 per burrito, and you will sell 300 burritos if you charge \$5.00 per burrito[4]. Write b as a linear function of p (I suggest you simply use the point-slope formula).

(d) Rewrite the net daily profit M that you found in part (b) as a function of p only, using your result from part (c). Your answer will be a function $M(p)$.

(e) How much should you charge per burrito in order to maximize your daily profits? What will your maximum daily profit be if you charge this amount?

[4] This is actually an unrealistically high number of sales per day. Figure that it takes you one minute to make a burrito—then you'll be working for five hours *straight* just to make all of the burritos you sell at \$5.00 each. Hope you had an assistant!—This is why you probably don't really want to abandon your science education to make burritos.