Calculus I
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
June 16, 2014

Week 4: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 3, 7, and 8 of Chapter 3 in time for Monday's lecture, and Sections 4, 5, and 6 of Chapter 3 in time for Wednesday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

True/False Study Guides

Please find at the end of each section, before the problems are given, the True/False Study Guide for that section. You should read through these true/false items to check your understanding of the section, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be relatively simple problems just for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 3.9, Problems 1–15 odd; 21–29 odd; 35, 45
- Section 3.10, Problems 1–20 odd; 27; 31; 39
- Section 4.2, Problems 17–33 odd; 47
- Section 4.3, Problems 1–6; 7, 9, 11–23 odd; 25, 29, 33, 37, 43

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[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ____________________

WPI Username: ________________

Week 4: Homework Problems

Due date: Friday, 20 June 2014, 11:59 p.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. A camera is located 5 m from a straight road, along which a car is travelling at 50 km/hr. The camera turns so that it is pointed at the car at all times.

(a) [**2 points**] Draw a picture of this scenario, choosing names for and labelling the relevant variables and the fixed quantity that you have been given in the problem statement.

(b) [**2 points**] Suppose that you are asked how fast the camera angle is changing when the car is closest to the camera. (Let $t = T$ represent the time when the car is closest to the camera.) Write down, using the variable $t = T$ and the other variables you chose in part (a), what you are being asked to find (do not do any computations now; just write down what you are looking for). Also in terms of those variables, write all of the information that the problem gives you (again, do not solve for anything). What is the camera angle at time $t = T$?

(c) [**2 points**] What is the relationship between the camera angle and the car's distance from the camera, as measured along the road? Write this out "in mathematics", *i.e.*, using the variables you chose in part (a), and be sure to include units.

(d) [**4 points**] Determine how fast the camera angle is changing when the car is closest to the camera, in units of radians per second. This will involve taking appropriate derivatives using implicit differentiation, and converting units before substituting them into your final expression. You are to show the appropriate units at *each* stage of your computation, and your final answer should also show units. Show *ALL* of your work.

Problem 2. This problem will show you how to find roots using Newton's method. See Equation 8 on Page 208 for the square root case.

(a) [**2 points**] Write down the iterative formula for Newton's method, taking $f(x) = x^3 - a$, where a is a constant. Show all of your work and simplify your expression as much as possible.

(b) [**2 points**] Write down the iterative formula for Newton's method, taking $f(x) = x^4 - a$, where a is a constant. Again, show all of your work and simplify your expression as much as possible.

(c) [**2 points**] Write down the iterative formula for Newton's method, taking $f(x) = x^k - a$, where a and k are both constants. I shouldn't have to tell you that you must show all of your work and simplify your expression as much as possible.

(d) [**4 points**] Use your formula from part (c) to find $\sqrt[9]{120}$ accurate to five decimal places. Write what you chose $f(x)$ to be, write the resulting iterative formula, write your initial guess x_0, and in general, show *ALL* of your work: you will receive absolutely no credit for giving only the final answer, and your solution must include equations to show how you obtained each of the successive approximations x_1, x_2, …, x_n.

Problem 3. The equation $x + \tan x = 0$ is important in a variety of applications—e.g., in the study of diffusion of heat.

(a) [**1 point**] For which values of x is the function $f(x) = x + \tan x$ continuous?

(b) [**2 points**] Prove, using the Intermediate Value Theorem, that $x + \tan x = 0$ has at least one solution inside the interval $[1.8, 2.2]$, and at least one solution in the interval $[4.8, 5.2]$.

(c) [**2 points**] Write the iterative formula for Newton's method using $f(x) = x + \tan x$. Show all work and simplify your expression as much as possible. Using this formula may be more convenient if you write it in terms of sine and cosine only.

(d) [**2 points**] Using this formula with the initial guess $x_0 = 1.9$, find a numerical approximation of the root within the interval $[1.8, 2.2]$, accurate to six decimal places. Again, you must show *all* of your work in order to receive credit.

(e) [**2 points**] Using this formula with the initial guess $x_0 = 4.9$, find a numerical approximation of the root within the interval $[4.8, 5.2]$, accurate to six decimal places. Again, you must show *all* of your work in order to receive credit.

(f) [**1 point**] Draw a graph of $f(x)$, including all axis labels, tick marks and arrows, noting the scale and clearly marking the positions of all asymptotes. Use the information you've found in the other parts of this problem; you may also wish to use a tool like WolframAlpha to assist you.