Calculus I
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
June 16, 2014

Week 6: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 4 and 5 of Chapter 4 in time for Monday's lecture, and Sections 6 and 7 in time for Wednesday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

True/False Study Guides

Please find at the end of each section, before the problems are given, the True/False Study Guide for that section. You should read through these true/false items to check your understanding of the section, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be relatively simple problems just for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 4.4, Problems 1–25 odd; 29, 41, 51
- Section 4.5, Problems 1–4; 5–25 odd; 49–53 odd
- Section 4.6, Problems 1–7 odd; 17–29 odd; 31–39 odd; 77–82
- Section 4.7, Problems 1–15 odd; 17–28; 29–49 odd

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: _____

WPI Username: _____

Week 6: Homework Problems

Due date: Friday, 27 June 2014, 11:59 p.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) No late assignments will be accepted for credit.

Problem 1. **(a)** [**5 points**] Prove that the equation $x^7 + x^5 + x^3 + 1 = 0$ has exactly one real solution. Carefully explain your reasoning, and if you refer to a function in your writeup, you must state how you have defined that function. You should be using two big theorems that we learned in class, and in order to do so, you must show explicitly how the hypotheses of each theorem are satisfied.

(b) [**5 points**] Show that the function $f(x) = x^{\frac{2}{3}}$ does *not* satisfy the hypotheses of the Mean Value Theorem over the interval $[-1, 27]$. Show that, nevertheless, there *does* exist a number $c \in (-1, 27)$ such that

$$f'(c) = \frac{f(27) - f(-1)}{27 - (-1)}.$$

Problem 2. Let $f(x) = x^{\frac{1}{3}}(6 - x)^{\frac{2}{3}}$.

(a) [**1 point**] Find the first derivative $f'(x)$ and the second derivative $f''(x)$.

(b) [**3 points**] Find and classify the critical points of f. Where is the tangent line horizontal, where is it vertical, and where does it fail to exist? Evaluate $f(x)$ at each of the critical points. Where is f increasing, and where is it decreasing?

(c) [**2 points**] Find the possible points of inflection (PPI) of f. Evaluate f at each of the PPI, and list which of the PPI are actual inflection points. Where is f concave up, and where is it concave down?

(d) [**1 point**] Find the x and y intercepts of f.

(e) [**3 points**] **Without the aid of calculators or computational tools**, sketch the graph $y = f(x)$.

Problem 3. Let $f(x) = \frac{2x^2+1}{x^2-2x}$.

(a) [**1 point**] Find the first derivative $f'(x)$ and the second derivative $f''(x)$.

(b) [**3 points**] Find and classify the critical points of f. Where is the line tangent to f horizontal, where is it vertical, and where does it fail to exist? Evaluate $f(x)$ at each of the critical points. Where is f increasing, and where is it decreasing?

(c) [**2 points**] Find the possible points of inflection of f. Evaluate f at each of the possible points of inflection, and list which of them are actually points of inflection. Where is f concave up, and where is it concave down?

(d) [**2 points**] Locate all vertical, horizontal, and/or slant asymptotes of f.

(e) [**2 points**] **Without the aid of calculators or computational tools**, sketch the graph $y = f(x)$.