Calculus II
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
Due May 23, 2016

Week 1: Reading, Practice Problems, and Homework Exercises

How This Works

Each week, the week's reading and homework problems will be posted to the course's myWPI page (`my.wpi.edu`). In mathematics, reading without working through problems is useless. So a number of interesting exercises are also assigned. Solving them is not usually hard, but writing them up neatly and meeting a deadline is less pleasant. So, while you are expected to look at all of the practice problems each week, your homework will only consist of only a few problems per week. These are intended to be more thought-provoking, and you are expected to complete these problems with great care and to present them in a professional way. You are encouraged to type your solutions using LaTeX, or to handwrite them very neatly and scan them at a sufficiently high resolution as to be legible. If your work is not presentable or if it is illegible, you will not receive credit for it.

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it. An example of acceptable homework solutions is posted on myWPI under "Course Materials".

Reading

Please read Sections 4.8 and 5.1–5.2 in time for Tuesday's lecture, and Sections 5.3 and 5.4 in time for Thursday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

Questions to Guide Your Review

Note: Do not hand these in!

Please find at the end of each chapter, before the chapter problems are given, the "Questions to Guide Your Review" section. You should read through these items to check your understanding of the chapter, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be conceptually important questions for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

The relevant questions for this week's material are:

- Chapter 4, "Questions to Guide Your Review", p. 291, Problems 21, 23, and 24
- Chapter 5, "Questions to Guide Your Review", p. 529, Problems 1–3 and 5–6

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

- Section 4.8, Problems 1; 7; 11–23 odd; 91–95 odd
- Section 5.1, Problems 1–7 odd; 15; 17
- Section 5.2, Problems 1–29 odd; 33–45 odd
- Section 5.3, Problems 1–13 odd; 37–47 odd; 51–61 odd
- Section 5.4, Problems 1–33 odd; 39–55 odd; 65–69

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Week 1: Homework Problems

Due date: Monday, May 23, 2015, 11:59 p.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) No late assignments will be accepted for credit.

Problem 1. In a vacuum, all bodies fall with the same constant acceleration. To demonstrate this, the Apollo 15 astronaut David Scott dropped a hammer and a feather on the moon from about 4 feet above the ground. The television footage of the event, which you can find at `https://commons.wikimedia.org/wiki/File:Apollo_15_feather_and_hammer_drop.ogg`, shows the hammer and the feather falling more slowly than they do on Earth, where, in a vacuum, they would have taken only half a second to fall 4 ft.

(a) [**8 points**] Solve the following initial value problem for s as a function of t.

$$\begin{cases} \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = -5.2 \text{ ft/sec}^2 \\ \left.\frac{\mathrm{d}s}{\mathrm{d}t}\right|_{t=0} = 0 \text{ ft/sec}, \quad s(0) = 4 \text{ ft.} \end{cases}$$

For the feather-and-hammer problem, $s(t)$ represents the position of the hammer and feather above the ground at time t, and $t = 0$ represents the time when Colonel Scott released the hammer and feather. Hint: you will solve the given initial value problem for $\frac{\mathrm{d}s}{\mathrm{d}t}$, and this will give you *another* initial value problem to solve for $s(t)$.

(b) [**2 points**] Call the time when the hammer and feather hit the ground t_{hit}. Find a value for t_{hit}, which satisfies $s(t_{\text{hit}}) = 0$.

Problem 2. [**6 points**] Suppose that the differentiable functions $y = F(x)$ and $y = G(x)$ both solve the initial value problem

$$\begin{cases} \frac{\mathrm{d}y}{\mathrm{d}x} = f(x), \\ y(x_0) = y_0 \end{cases}$$

for $x \in I$, an interval of real numbers.

(a) [**4 pt**] What is an initial value problem that the function $H(x) := F(x) - G(x)$ solves? [Hint: Compute $\frac{\mathrm{d}H}{\mathrm{d}x}$, and evaluate $H(x_0)$. Write in initial value problem form (using y as the "unknown" function).]

(b) [**2 pt**] Solve the initial value problem you found previously, to find out what $H(x)$ has to be. What does this mean about uniqueness of solutions to initial value problems?

Problem 3. [**5 pt**] What is the definite integral of $f(x) = x^3$ over the interval $[0, 1]$?

Problem 4. Using rectangles whose height is given by the value of the function at the *midpoint* of each rectangle, estimate the area under the graph of $f(x) = x^3$ between $x = 0$ and $x = 1$, using...

(a) [**4 pt**] Two rectangles

(b) [**5 pt**] Four rectangles

What is the error in each case? (That is, what is the absolute value of the definite integral minus the area estimate?)

Problem 5. [**15 pt**] Write a computer program in your language of choice[2] that uses the midpoint rule to approximate the area under the graph of $f(x) = x^3$ between $x = 0$ and $x = 1$, for $n = 100$, 500, and 1000 rectangles of equal length. What is the estimate of the area in each case, and what is the error in each case? You must show the entire text of your code, and show its output.

Problem 6. Write the following sums without sigma notation, and evaluate them.

(a) [**2 points**] $\displaystyle\sum_{k=1}^{3} \frac{k-1}{k}$

(b) [**2 points**] $\displaystyle\sum_{k=1}^{5} \sin(k\pi)$

(c) [**2 points**] $\displaystyle\sum_{k=1}^{2} \frac{6k}{k+1}$

Problem 7. Evaluate the following definite integrals.

(a) [**3 points**] $\displaystyle\int_0^2 x(x-3)\ \mathrm{d}x$

(b) [**3 points**] $\displaystyle\int_0^{\pi/4} \tan^2 x\ \mathrm{d}x$

(c) [**3 points**] $\displaystyle\int_0^{\ln 2} e^{3x}\ \mathrm{d}x$

[2] If you don't program yet, then I suggest MATLAB or Python, both straightforward languages that you can start with quickly. A free, in-browser Python interpreter can be found at `https://repl.it/languages/python3`, and a good introduction to scientific computing with Python can be found in the first chapter of `http://hplgit.github.io/primer.html/doc/pub/half/book.pdf` . **Please let me know early if you are stuck on this problem!**