Calculus II
E1 Term, Sections E101 and E196
Instructor: E.M. Kiley
Due June 20, 2016

Week 5: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it. An example of acceptable homework solutions is posted on myWPI under "Course Materials".

Reading

Please read Sections 7.2 and 8.1 in time for Tuesday's lecture, and Sections 8.2 and 8.3 in time for Thursday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

Questions to Guide Your Review

Note: Do not hand these in!

Please find at the end of each chapter, before the chapter problems are given, the "Questions to Guide Your Review" section. You should read through these items to check your understanding of the chapter, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be conceptually important questions for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

The relevant questions for this week's material are:

- Chapter 7, "Questions to Guide Your Review", p. 415, Problems
- Chapter 8, "Questions to Guide Your Review", p. 415, Problems

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 7.2, Problems 1–25 odd
- Section 8.1, Problems 1–39 odd
- Section 8.2, Problems 13–49 odd
- Section 8.3, Problems 1–61 odd

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Week 5: Homework Problems

Due date: Monday, June 20, 2015, 11:59 p.m. EDT. Please upload a `.pdf` version to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly using correct English.

III) Show your work. Explain your answers using **full English sentences**.

IV) No late assignments will be accepted for credit.

Problem 1. Currently, the average annual rate of interest on Vanguard's VBLTX (long-term bond index fund) is 7.67%.[2]

(a) [**5 points**] Suppose that you invest \$5,500 at one time this year in VBLTX, and that you stop contributing to this account afterward. If interest is compounded annually, and if the annual rate of interest stays fixed at 7.67% per year, how much money will you have in that account forty years from now?

(b) [**5 points**] Suppose that you wait ten years before investing your \$5,500. How much money will you have forty years from now in that case? (That is, how much will you have thirty years after you start investing?)

(c) [**5 points**] Suppose that beginning now, you invest \$5,500 per year in VBLTX for the next fifteen years (you deposit a total of fifteen times). How much money will you deposit over that period of time? (Do not compute interest yet.) If the interest rate remains fixed at 7.67% and interest is compounded annually, compute the interest to find out how much money will be in your account at the end of the fifteenth year.
Hint: The amount of money in your account will be the sum of fifteen terms:

$$\sum_{i=1}^{15} 5500 \cdot 1.0767^i = \underbrace{5500 \cdot 1.0767^1}_{\text{last deposit's contribution}} + 5500 \cdot 1.0767^2 + \cdots + 5500 \cdot 1.0767^{14} + \underbrace{5500 \cdot 1.0767^{15}}_{\text{first deposit's contribution}}.$$

You might find a tool like WolframAlpha useful in computing this sum; use this example as a template: `http://www.wolframalpha.com/input/?i=%5Csum_%7Bi%3D0%7D%5E%7B20%7D+43*1.06%5Ei`, and change the numbers in it to obtain the sum above (the sum is under the "Decimal Form" heading).

(d) [**5 points**] Continuing the scenario in part (c), suppose that you never make another contribution to that account after those first fifteen years go by, but that the interest rate remains fixed at the same rate of 7.67% and compounds annually for the next thirty years after. How much will be in that account after those next thirty years have passed?

(e) [**10 points**] Suppose that your friend never thinks very much about his retirement now, and after his mid-life crisis in fifteen years, he begins to invest \$5,500 per year in VBLTX, with the same fixed 7.67% rate of annually compounded interest, for the next thirty years after. How much money will he deposit in total? How much money will be in his account forty-five years from now, when you are both ready to retire? (Similar Hint: The total amount in his account at that time will be the sum of thirty terms—do this with WolframAlpha like you did part (c).)[3]

[2] The average is taken over all the years that Vanguard has had this fund available for customers to invest in—that is, since March 1994. You can see the return rates for some of Vanguard's other mutual funds here: `https://investor.vanguard.com/mutual-funds/vanguard-mutual-funds-list`. The returns on some of their funds are lower than those on VBLTX, and others are higher.

[3] I hope that doing this problem makes you think about how important it is to begin investing early. The student loan interest you pay won't be compounding for the rest of your life, but the interest your earn on your savings will—and, to boot, yearly IRA contributions are currently limited by U.S. law (to \$5,500 per year for people in your instructor's age and income bracket), making it truly impossible to catch up in the future if you fall behind now. So, learn more about IRAs here: `http://money.cnn.com/retirement/guide/IRA_Basics.moneymag/index.htm`, and begin investing while you're young (:

Problem 2. **[10 points]** One person at WPI has started a rumor about about Gompei the Goat. According to the WPI web site, the total number of undergraduate students, graduate students, and faculty/staff on campus is 6,535. Assume that the rumor spread can be modelled as exponential growth (and that the three groups of WPI community members mentioned above all talk to and among each other at the same rate). If in the first seven days, 20 people hear the rumor, how long will it take for the entire population to have heard the rumor?

Problem 3. **[10 points]** Solve the initial value problem

$$\begin{cases} \frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2y^2 - y^2, \\ y(0) = 1. \end{cases}$$

Problem 4. Evaluate the following indefinite integrals.

(a) **[5 points]** $\displaystyle\int e^{2x} \sin(3x)\ \mathrm{d}x$

(b) **[5 points]** $\displaystyle\int \frac{x}{1+x^4}\ \mathrm{d}x$