Lecture 6.

Announcements/ Assignments.

- Webwork 5 due Friday 11:59 p.m.
- Homework 2 ——— " ———.
- Webwork 6 due Monday 11:59 p.m.
- Homework 3 ——— " ———
- Midterm Exam Tuesday in class.
  - ONLINE STUDENTS: You have three options.

    (1) Come to campus to take the exam;

    (2) Wait until A-term to take the exam on campus;

    (3) Hire a proctoring service to take the exam ON TUESDAY FROM 6–8 p.m. EDT, near you.

  YOU __MUST__ CONTACT ME TO LET ME KNOW WHICH OPTION YOU SELECT!

Today

- 6.4: Areas of Surfaces of Revolution
- ~~6.6: Moments & Centers of Mass~~ (not this class)
- Midterm review

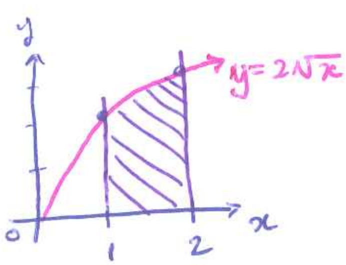
6.4: Areas of Surfaces of Revolution.

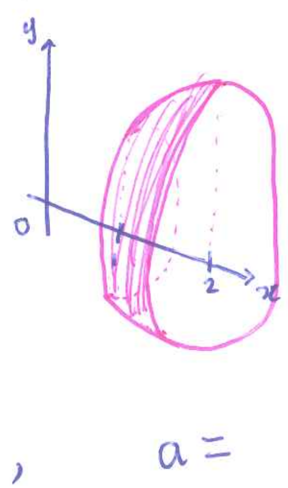
Last time, we used definite integrals to compute the volumes of surfaces of revolution — now, we'll compute the surface area.

> **DEFINITION:** If the function $f(x) \le 0$ is continuously differentiable on $[a,b]$, then the area of the surface generated by revolving the graph of $y = f(x)$ about the x-axis is:
>
> $$S = \int_a^b 2\pi f(x)\sqrt{1+[f'(x)]^2}\,dx = \int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Derivation on p. 391 of text.

Example 1, p.391 Find the area of the surface of revolution generated by revolving the curve $y = 2\sqrt{x}$, $1 \le x \le 2$, about the x-axis.





For our formula, $a =$

$b =$

$y =$

→

① Find $\frac{dy}{dx}$. $\frac{dy}{dx} = \frac{d}{dx}\left[2\sqrt{x}\right] = \frac{d}{dx}\left[2x^{1/2}\right] =$

② Find $\sqrt{1+\left(\frac{dy}{dx}\right)^2}$.

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2} = \sqrt{1+\frac{1}{x}} = \sqrt{\frac{x+1}{x}}.$$

③ Integrate $2\pi y \cdot \sqrt{1+(dy/dx)^2}$

$$S = \int_1^2 2\pi\,(2\sqrt{x})\sqrt{\frac{x+1}{x}}\;dx$$

$$= 4\pi \int_1^2 \sqrt{x}\left(\frac{\sqrt{x+1}}{\sqrt{x}}\right) dx$$

Last time, we also computed volumes of solids generated by revolving about the y-axis.

Surface Area Analogue:

If: $x = g(y) \geq 0$ is continuously differentiable on $[c, d]$, ~~then~~

Then: the area of the surface generated by revolving the graph of $x = g(y)$ about the y-axis is:

$$S = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2}\, dy = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy.$$

Example 2, p. 392 The line segment $x = 1 - y$, $0 \leq y \leq 1$, is revolved about the y-axis to generate a cone. Find its lateral surface area (excluding the base area).

Apply formula: $c =$ $\quad$ $x =$

$d =$

So, $S = \int_c^d 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Midterm Exam: Review.

- Lecture 1: 4.8 (Antiderivatives)
  - 5.1 (Finite Sums)
  - 5.2 (Area under curves)
  - 5.3 (Riemann sums & the Definite Integral)
- Lecture 2: 5.4 (Fundamental Thm. of Calculus)
- Lecture 3: 5.5 (Substitution for Indefinite Integrals)
- Lecture 4: 5.6 (Substitution for Definite Integrals)
- Lecture 5: 6.1 (Volumes using cross-sections)
  - 6.3 (Arc Length)
- Lecture 6: 6.4 (Surface Area of Solids of Revolution)

- Exam will be two hours long
- No calculators
- One sheet of notes will be allowed

- 90 total points (30% of course grade)

How to study for an exam?

- DO THE PRACTICE PROBLEMS: Homework
  Webwork
  Lecture notes/textbook
  Suggested problems