Calculus III
E2 Term, Sections E201 and E296
Instructor: E.M. Kiley
Due Wednesday, August 05, 2015, 11:59 p.m. EDT

Week 4: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 11.1, and 11.2 in time for Thursday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

Questions to Guide Your Review

Note: Do not hand these in!

Please find at the end of each chapter, before the chapter problems are given, the "Questions to Guide Your Review" section. You should read through these items to check your understanding of the chapter, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be conceptually important questions for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

The relevant questions for this week's material are:

- Chapter 11, "Questions to Guide Your Review", p. 647, Problems 1, 2, 4–6, 8

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 11.1, Problems 1–37 odd
- Section 11.2, Problems 1–35 odd

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ______________________

WPI Username: ______________

Week 4: Homework Problems

Due date: Wednesday, August 5, 2015, 11:59 p.m. EDT. Please upload a single `.pdf` document to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly.

III) **Show your work**. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. Find a parametrization for the following curves:

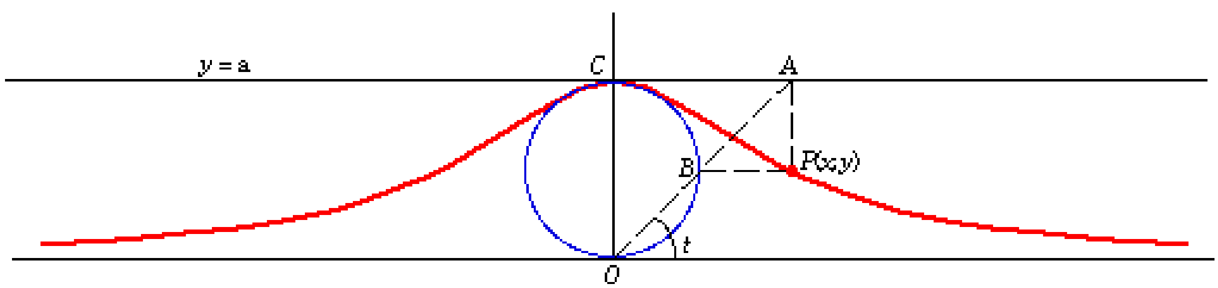
(a) [**2 points**] The line segment with endpoints $(-1, -3)$ and $(3, 2)$.

(b) [**2 points**] The ray (half line) with initial point $(2, 3)$ that also passes through the point $(-1, 1)$.

(c) [**3 points**] The lower half of the parabola $x - 2 = 2y^2$.

(d) [**3 points**] The left-hand half of the parabola $y = x^2 + 2x$.

Problem 2. [**10 points**] The bell-shaped witches of Maria Agnesi[2] form a set of curves; we will construct one of them in the following way. Starting with a circle of radius 1 and centered at the point $(0, 1)$, choose a point A on the line $y = 2$ and connect it to the origin with a line segment. Call the point where the segment crosses the circle B, and let P be the point where the vertical line through A crosses the horizontal line through B. The witch[3] is the curve traced by P as A moves along the line $y = 2$:



Find parametric equations and a parameter interval for the witch by expressing the coordinates of P in terms of t, the radian measure of the angle that the segment OA makes with the positive x-axis. It may help if you use the following inequalities (which you do not have to prove):

$$x = AC, \qquad y = 2 - AB \sin t, \qquad AB \cdot OA = (AC)^2.$$

Remember that on this and all homework problems, **you must show your work**. If you write down the parametric equations with no explanation, you will get zero credit.

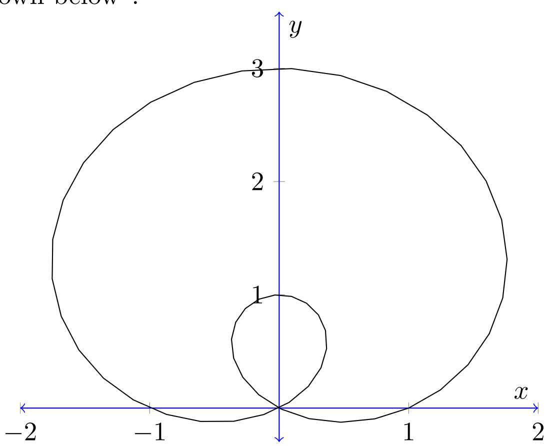
[2] Agnesi was an interesting figure in mathematics; the curious can read a little about her life here: `https://en.wikipedia.org/wiki/Maria_Gaetana_Agnesi` .

[3] For a `.gif` of this construction process, see `http://mathworld.wolfram.com/images/gifs/WitchOfAgnesi.gif` .

Problem 3. The curve with parametric equations

$$x = (1 + 2\sin\theta)\cos\theta, \qquad y = (1 + 2\sin\theta)\sin\theta$$

is called a *limaçon* and is shown below[4].



Find the points (x, y) and the slopes of the tangent lines at these points for the following values of the parameter θ:

(a) **[3 points]** $\theta = 0$.

(b) **[3 points]** $\theta = \pi/2$.

(c) **[4 points]** $\theta = 4\pi/3$. For this θ value, please also write an equation for the line tangent to the limaçon at the point (x, y). (The equation of the tangent line can be in Cartesian coordinates—no need to parametrize.)

Problem 4. To illustrate the fact that the numbers we get for curve length do not depend on the way we parametrize our curves (except for the restrictions preventing doubling back mentioned in the definition of the arc length), calculate the length of the semicircle $y = \sqrt{1 - x^2}$ with these two different parametrizations:

(a) **[5 points]** $x = \cos(2t), \quad y = \sin(2t), \quad t \in [0, \pi/2]$.

(b) **[5 points]** $x = \sin(\pi t), \quad y = \cos(\pi t), \quad t \in [-1/2, 1/2]$.

Problem 5. **[10 points]** The line segment joining the origin to the point (h, r) is revolved about the x-axis to generate a cone of height h and base radius r. Find the cone's surface area with the parametric equations $x = ht$, $y = rt$, $0 \le t \le 1$. Check your result against the standard formula: $A = \pi r \cdot (\text{slant height})$.

Problem 6. **[10 points]** Find the area under one arch of the cycloid

$$x = a(t - \sin t), \qquad y = a(1 - \cos t).$$

[4] A `.gif` of the process of drawing a limaçon is here: `https://en.wikipedia.org/wiki/File:PedalCurve2.gif`.