Calculus III
E2 Term, Sections E201 and E296
Instructor: E.M. Kiley
Due Monday, August 10, 2015, 11:59 p.m. EDT

Week 5: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 11.3, 11.4, and 11.5 in time for Tuesday's lecture, and 12.1, 12.3, 12.3, and 12.4 in time for Thursday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

Questions to Guide Your Review

Note: Do not hand these in!

Please find at the end of each chapter, before the chapter problems are given, the "Questions to Guide Your Review" section. You should read through these items to check your understanding of the chapter, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be conceptually important questions for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

The relevant questions for this week's material are:

- Chapter 11, "Questions to Guide Your Review", p. 699, Problems 9–13
- Chapter 12, "Questions to Guide Your Review", p. 745, Problems 1–10

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 11.3, Problems 1–23 odd; 41–61 odd
- Section 11.4, Problems 1–31 odd
- Section 11.5, Problems 1–29 odd
- Section 12.1, Problems 1–27 odd; 35–49 odd
- Section 12.2, Problems 1–39 odd
- Section 12.3, Problems 1–25 odd
- Section 12.4, Problems 1–27 odd

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ____________________

WPI Username: ____________________

Week 5: Homework Problems

Due date: Monday, August 10, 2015, 11:59 p.m. EDT. Please upload a single `.pdf` document to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly.

III) **Show your work**. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. Replace the Cartesian equations with equivalent polar equations. Please use Section 11.3 as reference.

(a) [**2 points**] $x = a$, where a is a constant. (The Cartesian equation is a vertical line.)

(b) [**2 points**] $y = b$, where b is a constant. (The Cartesian equation is a horizontal line.)

(c) [**3 points**] $x^2 + xy + y^2 = 1$.

(d) [**3 points**] $(x + 2)^2 + (y - 5)^2 = 16$.

Problem 2. Graph the following *limaçons*, without using graphing software or computational aids. Please see Section 11.4.

(a) [**2 points**] $r = \frac{1}{2} + \cos\theta$ (this limaçon has an inner loop).

(b) [**2 points**] $r = 1 - \cos\theta$ (this limaçon looks like a cardioid).

(c) [**3 points**] $r = \frac{3}{2} + \cos\theta$ (this limaçon has a dimple instead of a loop).

(d) [**3 points**] $r = 2 + \cos\theta$ (this limaçon looks like an oval).

Problem 3. These problems are about finding the areas inside and between polar curves. Please see Section 11.5.

(a) [**5 points**] Find the area inside the leminiscate $r^2 = 6\cos(2\theta)$ and outside the circle $r = \sqrt{3}$.

(b) [**5 points**] Find the area inside the six-leaved rose $r^2 = 2\sin(3\theta)$.

Problem 4. [**10 points**] Vectors are drawn from the center of a regular[2] n-sided polygon in the plane to the vertices of the polygon. Show that the sum of the vectors is zero. [Hint: What happens to the sum of the vectors if you rotate the polygon about its center?]. **You must give a well-structured and coherent logical argument, using sentences, to justify your answer. Your answer should not just be a string of calculations or a mess of arrows and diagrams. Also, an example does not constitute a general proof: do not just show this for a 3-sided or 4-sided polygon!**

[2] A regular polygon is one that has all angles equal in measure, and all sides equal in length. Like an equilateral triangle (a regular 3-sided polygon), or a square (a regular 4-sided polygon).

Problem 5. **[10 points]** This problem is about the famous Cauchy-Schwartz inequality. Here, we will prove this inequality in $\mathbb{R}^3$, but it holds in any vector space[3].

(a) **[5 points]** Since $\vec{u} \cdot \vec{v} = |\vec{u}|\,|\vec{v}|\cos\theta$, where θ is the angle between $\vec{u}$ and $\vec{v}$, show that the inequality $|\vec{u} \cdot \vec{v}| \leq |\vec{u}|\,|\vec{v}|$ holds for any vectors $\vec{u}$ and $\vec{v}$.

(b) **[5 points]** Under what circumstances, if any, does $|\vec{u} \cdot \vec{v}|$ equal $|\vec{u}|\,|\vec{v}|$? You must justify your answer.

Problem 6. Which of the following are *always true*, and which are *not always true*? Give reasons for your answers.

(a) **[2 points]** $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

(b) **[2 points]** $\vec{u} \times \vec{v} = \vec{v} \times \vec{u}$

(c) **[2 points]** $c(\vec{u} \times \vec{v}) = (c\vec{u}) \times \vec{v} = \vec{u} \times (c\vec{v})$ for any number c

(d) **[2 points]** $\vec{u} \cdot \vec{u} = |\vec{u}|^2$

(e) **[2 points]** $(\vec{u} \times \vec{v}) \cdot \vec{u} = \vec{v} \cdot (\vec{u} \times \vec{v})$

[3] Read more about the inequality here: `https://en.wikipedia.org/wiki/Cauchy\%E2\%80\%93Schwarz_inequality` .