Calculus III
E2 Term, Sections E201 and E296
Instructor: E.M. Kiley
Due Wednesday, August 19, 2015, 11:59 p.m. EDT

Week 6: Reading, Practice Problems, and Homework Exercises

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use any symbols, you should do so *only* in full sentences where you intend to abbreviate words.

If the work that you submit is incomplete or illegible, you will not receive credit for it.

Reading

Please read Sections 12.5 and 13.1 in time for Tuesday's lecture, and 13.2, 13.3, and 13.4 in time for Thursday's lecture. (In-class students, you can always re-watch the lectures online after you finish your reading, if it would benefit you.) I will not necessarily cover all of this material in class, but you will be responsible for it. Any questions about any of the material can be addressed in class or office hours, or to me via e-mail (`emkiley@wpi.edu`).

Questions to Guide Your Review

Note: Do not hand these in!

Please find at the end of each chapter, before the chapter problems are given, the "Questions to Guide Your Review" section. You should read through these items to check your understanding of the chapter, but you are not required to hand in your answers. If you have questions about these, you will usually be able to find your answer by re-reading the section, by consulting the hints in the back of the book, or, if you are really stuck, by consulting me. These are meant to be conceptually important questions for you to check how well you have understood the material in each section, and if you expect to do well on the midterm and final exams, I suggest studying these in particular.

The relevant questions for this week's material are:

- Chapter 12, "Questions to Guide Your Review", p. 745, Problems 9, 11, 12
- Chapter 13, "Questions to Guide Your Review", p. 788, Problems 1–11

Practice Problems

Note: Do not hand these in!

Here are some practice problems to work on at home. It is extremely important that you are proficient at exercises such as these; without the basic skills, you will find it difficult to complete your exams in the allotted time.

You will find the answers to the odd-numbered problems in the back of the book. This is useful if you want to check your work, but please remember that the *logical argument*, not the final answer, is the most important part of solving a problem for credit in this class. You should therefore understand *how to solve* each of these problems. In particular, you should *not* be satisfied with merely looking up the solution in the back of the book.

Please discuss any questions with me in class, during my office hours, or send me an e-mail.

- Section 12.5, Problems 1–17 odd; 21–59 odd
- Section 13.1, Problems 1–23 odd
- Section 13.2, Problems 1–37 odd
- Section 13.3, Problems 1–15 odd
- Section 13.4, Problems 1–17 odd

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

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Name: ____________________

WPI Username: ____________________

Week 6: Homework Problems

Due date: Wednesday, August 19, 2015, 11:59 p.m. EDT. Please upload a single `.pdf` document to myWPI (`my.wpi.edu`).

Rules for Calculus Assignments:

I) Each student must compose his or her assignments independently. However, brainstorming may be done in groups.

II) Please typeset your solutions using LaTeX, or handwrite them neatly and legibly.

III) **Show your work**. Explain your answers using **full English sentences**.

IV) **No late assignments will be accepted for credit.**

Problem 1. [**10 points**] Prove that four points A, B, C, and D are coplanar (i.e., they lie in a single common plane) if and only if $\overrightarrow{AD} \cdot \left(\overrightarrow{AB} \times \overrightarrow{BC}\right) = 0$. [Hint: Remember that there are two things to prove for an "if and only if" statement: the forward direction of the implication, and also the backward direction.]

Problem 2. Suppose that a particle moves in the xy-plane in such a way that its position at time t is given by

$$\vec{r}(t) = (t - \sin t)\,\hat{\imath} + (1 - \cos t)\,\hat{\jmath}.$$

(a) [**3 points**] Graph $\vec{r}(t)$. [Hint: This curve is called a cycloid, and you should be able to graph it without help, but you may use a graphing calculator or computational aid if you wish.]

(b) [**7 points**] Find the maximum and minimum values of $|\vec{v}|$ and $|\vec{a}|$, where $\vec{v}$ and $\vec{a}$ represent the velocity and the acceleration, respectively, of the particle. [Hint: Find the extrema of $|\vec{v}|^2$ and $|\vec{a}|^2$ first, and then take square roots later.]

Problem 3. [**7 points**] An electron in a cathode ray tube is beamed horizontally at a speed of 5×10^6 meters per second toward the inside face of a television 40 cm away. About how far will the electron drop before it hits?

Problem 4. A volleyball is hit when it is 4 feet above the ground and 12 feet from a 6-foot-high net. It leaves the point of impact with an initial velocity of 35 feet per second, at an angle of 27° from the horizontal, and slips by the opposing team untouched.

(a) [**2 points**] Find a vector equation for the path of the volleyball.

(b) [**3 points**] How high does the volleyball go, and when does it reach maximum height?

(c) [**3 points**] Find its range and flight time.

(d) [**3 points**] When is the volleyball 7 feet above the ground? How far (ground distance) is the volleyball from where it will land?

(e) [**2 points**] Suppose the net is raised to 8 feet. Does this change things? Explain your answer.

Problem 5. To illustrate that the length of a smooth space curve does not depend on the parametrization you use to compute it, calculate the length of one turn of the same helix with the three different parametrizations:

(a) [**4 points**] $\vec{r}(t) = (\cos 4t)\hat{\imath} + (\sin 4t)\hat{\jmath} + 4t\hat{k}, \quad 0 \le t \le \pi/2.$

(b) [**3 points**] $\vec{r}(t) = (\cos \frac{t}{2})\hat{\imath} + (\sin \frac{t}{2})\hat{\jmath} + \frac{t}{2}\hat{k}, \quad 0 \le t \le 4\pi.$

(c) [**3 points**] $\vec{r}(t) = (\cos t)\hat{\imath} - (\sin t)\hat{\jmath} - t\hat{k}, \quad -2\pi \le t \le 0.$

Problem 6. [**10 points**] Show that the parabola $y = ax^2$, where $a \neq 0$, has its largest curvature at its vertex and has no minimum curvature. [Note: Since the curvature of a curve remains the same if the curve is translated or rotated, this result is true for any parabola.]