Lecture 10:
Lesson and Activity Packet

MATH 232: Introduction to Statistics

October 24, 2016

Homework and Announcements

- Complete Book Choice Survey on Canvas (this is your Homework 11 grade)
- Election Prediction Project
  - Friday will be a project day
  - Deadline for ASA submission is Sunday at 8 p.m.
  - Deadline for group report, proof of ASA submission, and individual report is Monday at 11:59 p.m.
  - Please let me know before Sunday whether your group wants to present in class on Monday
- Quiz 6 on Wednesday (conditional probability)
- Homework 12 due in class on Friday (find it on Canvas Tuesday morning)

Questions?

Recap of today

- Conditional probability

We now introduce **conditional probability**.
Suppose that we toss two dice, and suppose that each of the possible 36 outcomes is equally likely to occur. Suppose further that we observe the first die is a 3 (the second one lands under the table, so we can't see it yet).

Given this information, what is the probability that the sum of the two dice equals 8?

... well, there's only one combination, $(3,5)$, out of the six possible combinations starting with 3 : $\{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$,

so $P\big((3,5)\big) = \frac{1}{6}$.

If we let E and F, respectively, denote the event that the sum of the dice is 8 and the event that the first die is 3, then the probability you just computed is called the **conditional probability** that E occurs given that F has occurred, and is denoted by the expression

$$P(E|F).$$

A general formula for $P(E|F)$ that is valid for all events E and F is derived in the same manner: If the event F occurs, then in order for E to occur it is necessary that the actual occurrence be a point in both E and in F.

Definition 1

If $P(F) > 0$, then

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)}$$

Revisiting the example of the dice summing to 8:

Group Exercise 1

When rolling two dice, what are the outcomes that correspond to the first die landing on 3? List them in a set called F. What is $P(F)$?

$$F = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$$

$P(F) = \frac{6}{36}$, because there are $6 \cdot 6 = 36$ outcomes in the sample space of tossing two dice.

Group Exercise 2

When rolling two dice, what are the outcomes that lead to a sum of 8? List them in a set called E.

$$E = \{(2,6), (6,2), (4,4), (3,5), (5,3)\}$$

Group Exercise 3

What is the intersection of E and F? That is, write down $E \cap F$. What is $P(E \cap F)$?

$E \cap F = \{(3,5)\}$. $\qquad P(E \cap F) = \frac{1}{36}$.

Group Exercise 4

Using the formula, what is $P(E|F)$?

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/36}{6/36} = \frac{1}{36} \cdot \frac{36}{6} = \frac{1}{6}.$$

Example 1

A coin is flipped twice. Assuming that all four points in the sample space are equally likely, what is the conditional probability that both flips land on heads, given that the first flip lands on heads?

$S = \{(H,H), (H,T), (T,H), (T,T)\}$.

$F = \{(H,H), (H,T)\}$, so $P(F) = \frac{2}{4}$

$E = \{(H,H)\}$, so $E \cap F = \{(H,H)\}$ and $P(E \cap F) = \frac{1}{4}$.

$$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{2/4} = \frac{1}{4} \cdot \frac{4}{2} = \frac{1}{2}.$$

Group Exercise 5

What is the conditional probability that both flips land on heads, given that at least one flip lands on heads?

S and E are same

$F = \{(H,H), (H,T), (T,H)\}$, so $P(F) = 3/4$.

$E \cap F = \{(H,H)\}$ so $P(E \cap F) = 1/4$

Thus, $$P(E|F) = \frac{P(E \cap F)}{P(F)} = \frac{1/4}{3/4} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}.$$

Group Exercise 6

Three cards are randomly selected, without replacement, from an ordinary deck. Compute the conditional probability that the first card selected is a spade, given that the second and third cards are spades.

If 2 cards are spades, then there are ~~13~~ 11 more spades to choose from (out of the original 13), and if we've taken 2 cards from the deck already, then there are 50 cards left to choose from.

So $P(1^{st} \text{ is spade} \mid 2^{nd} \,\&\, 3^{rd} \text{ are}) = \frac{11}{50}$.

An important note: don't confuse $P(E|F)$ with $P(F|E)$. For example, in Massachusetts, the probability that it is dark outdoors, given that it is midnight, is 1: $P(\text{dark}|\text{midnight}) = 1$. But the probability that it is (exactly) midnight given that it is dark is very small, because it's dark for a long time each night, but it's only midnight for an instant. So $P(\text{midnight}|\text{dark}) \approx 0$.