Lecture 12:
Lesson and Activity Packet

MATH 232: Introduction to Statistics

October 31, 2016

Homework and Announcements

- Homework 13 due in class on Friday (find it posted on Canvas tonight)
- Election Prediction Project
  - Deadline for ASA submission **was yesterday**
  - Deadline for group report, proof of ASA submission, and individual report is **tonight** at 11:59 p.m.
- Quiz 6 on Wednesday will cover everything through today
- Exam 2 is one week from Wednesday!
  - It will cover everything we do this week.
  - Monday (one week from today) will be a review day.
- Book Review Project
  - Details to be given after the Election Project, **but** you should buy or procure (by legal means. . .) the book of your choice now, from among the following three choices:
    1. *Moneyball: The Art of Winning an Unfair Game*, by Michael Lewis
    2. *Dataclysm: Love, Sex, Race, and Identity—What Our Online Lives Tell Us about Our Offline Selves*, by Christan Rudder
    3. *How Not to Be Wrong*, by Jordan Ellenberg
  - You must make your choice before Monday, and **use groups on Canvas to sign up for your desired book**

Last time

- Conditional probability
- Independent events
- Multiplication rules

Recap of today

- More independent events and multiplication rules
- Bayes' Theorem (?)

Last time, we had the definition of *independent events*:

Definition 1 (*Independent Events*)

If $P(B) \neq 0$ and if $P(A|B) = P(A)$, we say that event A is independent *of event B. That is, event A is independent of event B if the probability of A is not affected by the occurrence or non-occurrence of event B.*

Group Exercise 1

The probabilities that it will rain or snow in a given city on Christmas Day, on New Year's Day, or on both days, are $P(C) := 0.60$, $P(N) := 0.60$, and $P(C \cap N) := 0.42$. Check whether N and C are independent.

Group Exercise 2

If $P(A) = 0.80$, $P(C) = 0.95$, and $P(A \cap C) = 0.76$, are events A and C independent?

Some interesting Multiplication Rules follow from the definition of the conditional probability of E given F:

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)}.$$

If we multiply both sides of this equation by $P(F)$, we obtain one version of the general multiplication rule:

Theorem 1

General multiplication rule

$$P(E \text{ and } F) = P(F) \cdot P(E|F)$$

But also, the definition of the conditional probability of F given E is:

$$P(F|E) = \frac{P(F \text{ and } E)}{P(E)}.$$

Noticing that for all events A and B, $P(A \text{ and } B) = P(B \text{ and } A)$, the conditional probability of F given E becomes

$$P(F|E) = \frac{P(E \text{ and } F)}{P(E)},$$

and we can again multiply both sides of the equation by $P(E)$ to obtain **another** version of the general multiplication rule:

Theorem 2

General multiplication rule

$$P(E \text{ and } F) = P(E) \cdot P(F|E).$$

Both of these rules hold for **all events** E and F.

Example 1

A jury consists of nine men and three women. If two of the jurors are randomly picked for an interview, what is the probability that they will both be women?

Let A be the event that the first juror is a woman, and B be the event that the second juror is a woman. The probability we seek is $P(A \text{ and } B)$.

Because we are selecting jurors at random, the probability that the first juror picked will be a woman is $P(A) =$________.

If the first juror is already picked and ends up being a woman, then we have 11 jurors left to choose from, two of whom will be women. This means that $P(B|A) =$________. ***[Notice: we didn't need to use the formula for conditional probability at this stage of the problem—just our general knowledge of classical probability. This is okay, and in fact, it's what's intended!]***

We then use the multiplication rule to compute $P(A \text{ and } B)$:

$$P(A \text{ and } B) = P(A) \cdot P(B|A) =$$

If A and B are independent events, then recall that this means $P(B|A) = P(B)$, and $P(A|B) = P(A)$. For two events that are **already known to be independent**, the general multiplication rule then boils down to:

Theorem 3 (*General multiplication rule for independent events*)

$$P(A \text{ and } B) = P(A) \cdot P(B|A) = P(A) \cdot P(B).$$

Example 2

What is the probability of getting heads in two flips of a balanced coin?

Since the two events are independent, and the probability of heads is $\frac{1}{2}$ for each flip, the answer is $\frac{1}{2} \cdot 12 = \frac{1}{4}$.

Example 3

Revisiting the snowing-on-Christmas example: The probabilities that it will rain or snow in a given city on Christmas Day, on New Year's Day, or on both days, are $P(C) := 0.60$, $P(N) := 0.60$, and $P(C \cap N) := 0.42$. Check whether N and C are independent.

Example 4

If A and B are independent events and $P(A) = 0.20$ and $P(B) = 0.45$, find:

(a) $P(A \text{ and } B)$

(b) $P(A|B)$

(c) $P(B|A)$

(d) $P(A^C \text{ and } B)$

(e) $P(A \text{ and } B^C)$

(f) $P(B^C|A^C)$

(g) $P(A^C \text{ and } B^C)$

(h) $P(A \text{ or } B)$

Group Exercise 3

What is the probability of getting three heads in three flips of a balanced coin?

Group Exercise 4

A jury consists of nine men and three women. If ***three*** *of the jurors are randomly picked for an interview, what is the probability that they will all be women?*

Remember the example about $P(\text{dark}|\text{midnight}) \neq P(\text{midnight}|\text{dark})$.

Group Exercise 5

Suppose that C is the vent that a person committed a crime, and E is the event that a particular piece of evidence was found. State in words what $P(E|C)$ and $P(C|E)$ represent. How might this information be misunderstood by the general public?

Although these examples show us that $P(A|B) \neq P(B|A)$, we might be interested in the question of whether there is some other formula that relates $P(A|B)$ and $P(B|A)$.
Recall the two versions of the multiplication rule:

$$P(E \text{ and } F) = P(F) \cdot P(E|F)$$

and

$$P(E \text{ and } F) = P(E) \cdot P(F|E).$$

Notice that these two version of the rule give two different ways of writing the same quantity $P(E \text{ and } F)$. So we can write:

$$P(F) \cdot P(E|F) = P(E) \cdot P(F|E),$$

which boils down to

Theorem 4 (*Bayes' Theorem*)

$$P(E|F) = \frac{P(E) \cdot P(F|E)}{P(F)}.$$