Lecture 17:
Lesson and Activity Packet

MATH 232: Introduction to Statistics

December 5, 2016

Homework and Announcements

- Quiz 10 on Wednesday, December 7
- Last class on Monday, December 12 (one week from today)
- Homework 17 due in class Monday, December 12
- Extra Credit Opportunity due on Canvas at 11:59 p.m. on Monday, December 12 (you may also submit in hard copy during class that day)
- Post on book discussion forum on Canvas before 11:59 p.m. Monday
- Submit book summary 3 on Canvas before 11:59 p.m. Monday
- 10 a.m. section: your final exam is December 14, 10:30 a.m. in the usual classroom
- 12 p.m. section: your final exam is December 16, 1:00 p.m. in the usual classroom

Last time:

- Continuous random variables
- Normal distribution
- Area as probability

Questions?

Today

- Normal distribution

Recall that the requirements of a density curve are that the area beneath the curve sum to 1, and that the curve never fall below the x-axis.

Group Exercise 1

Verify that $f(x) := \frac{x}{8}$ *can serve as the probability density function for a continuous random variable that can take on any value from 0 to 4.*

Further, recall that we used the uniform probability distribution to illustrate the notion of a correspondence between **area beneath the curve** and **probability**.

Definition 1 (*Probability*)

The area under the density curve between any two values a *and* b *gives the probability that a random variable having the continuous distribution corresponding to that density curve will take on a value on the interval from* a *to* b.

Group Exercise 2

With reference to the preceding exercise, find the probabilities that a random variable having the given probability density will take on a value:

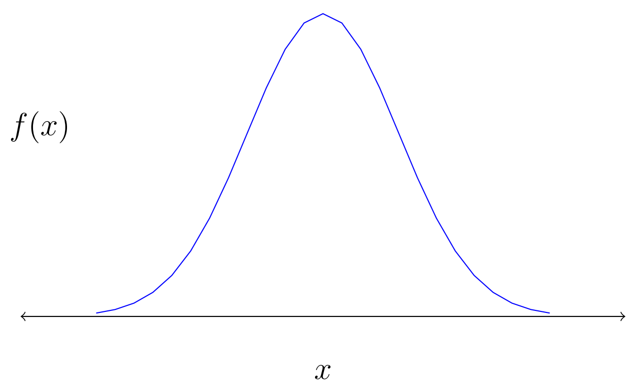
- *less than 2;*
- *less than or equal to 2*

We learned last time about the uniformly distributed continuous random variables. This time, we'll learn about normally distributed continuous random variables.

Definition 2 (*Normal random variable*)

If a continuous random variable has a distribution with a graph that is symmetric and bell-shaped, as in the figure below, and if this bell shape can be described by the equation below, then we say that the random variable is normally distributed *or that it follows the* normal distribution.

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$



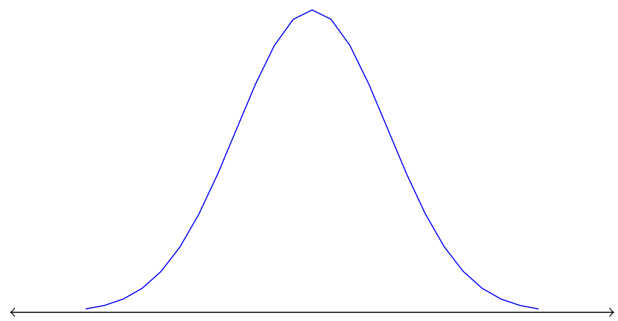
You do not need to memorize this formula, but look at it for a moment. It involves π and e, two irrational constants (their approximate values are 3.1415... and 2.71828..., respectively), and it involves σ and μ, the mean and standard deviation, respectively, of the random variable.

We see that the shape of the bell curve is entirely defined by the mean and standard deviation of the random variable it describes. Once we know μ and σ, we can draw the graph just like we would any other graph.

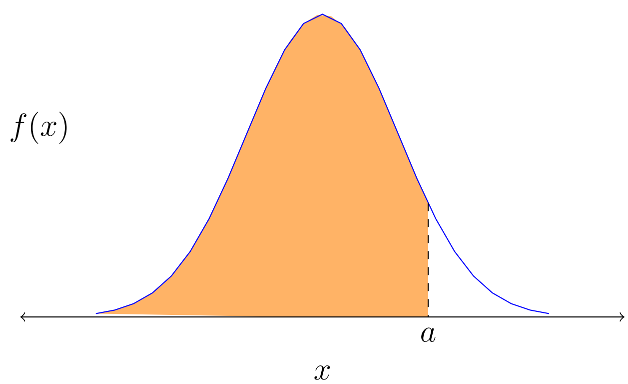
Note that the "tails" of the curve to to $\pm\infty$. There is **no end** to the range of values that this continuous random variable can take on, and that is okay: the area beneath the curve is still only 1. We will not prove this here, since it takes a great deal of mathematics to show.

Definition 3 (***Standard normal distribution***)

The standard normal distribution is the normal probability distribution with $\mu = 0$ *and* $\sigma = 1$*:*



Recall that for continuous random variables, the probability that $x = a$, for any a in the domain of x, is exactly 0. But we **do** have a way of defining the probability that $x < a$, that $x > a$, or that $a < x < b$. Just compute the area under the curve in that range. For example, the probability that $x < a$ or that $x \leq a$ is given by the shaded area:



Without the tools of calculus and analysis, there is no way of computing this area directly; however, there is a chart that you've been given on a separate piece of paper that gives a table of values that are useful for this. Some useful guidance and examples follow.