Lecture 8:
Lesson and Activity Packet

MATH 232: Introduction to Statistics

October 12, 2016

Homework and Announcements

- Exam 1 Redux: This comes as activity 1 in today's packet, but briefly:
  - min = 30
  - $Q_1 = 56.5$
  - $Q_2 = 76$
  - $Q_3 = 91.25$
  - max = 105
  - $\sigma = 21$
- Arithmetic on final course grades
- Homework 8 due in class on Friday (will be posted to Canvas later today)
- Election Prediction Project
  - Must sign up for groups on Canvas (see e-mail and announcement about this) **before Friday**
  - Friday will be a project day

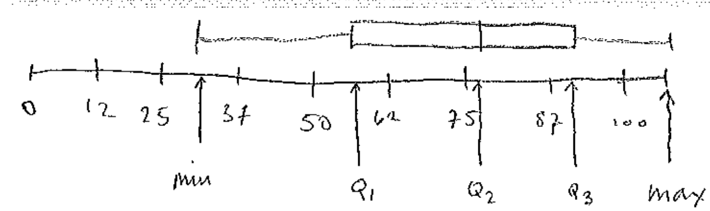
Questions?

The exam results were as follows:

- min = 30
- $Q_1 = 56.5$
- $\mu = 73.55$
- $Q_2 = 76$
- $Q_3 = 91.25$
- max = 105
- $\sigma = 21$

Group Exercise 1 ***(4 minutes)***

1. *Draw a box plot representing the exam results.*
2. *Formulate one statement using statistical concepts that explains or interprets the results.*



- The median is more than halfway across the IQR, so a higher concentration of scores in the range (76, 91.25).
- Half of scores are btwn. 57 and 91.25

Solutions are posted to Canvas. Any questions?
If not, then we'll continue with probability.

Recall that we learned the definition of the **complement** of an event:

Definition 1 (*Complement*)
For an event E, the complement of that event, denoted E^C, is everything in the sample space that is not in E.

We also learned that $P(E) + P(E^C) = 1$.

Example 1
On Exam 1, we were given that the probability of Donald Trump winning the presidential election in Mississippi was 86 %, and we were asked what Hillary Clinton's probability of winning that state was, assuming that no other candidate besides Clinton or Trump could win.

Question 1
Why is it important to assume that Trump and Clinton are the only ones in the race? What is the probability that Clinton will win Mississippi?

Some of you may have seen Venn diagrams before. If the box S represents the sample space and E represents an event in S, then the Venn Diagram shows that E^C is exactly the portion of S that is not part of E.

Q1(a): It's important to know that of the remaining vote (the 14% that doesn't go to Trump), all goes to Clinton.

Q1(b): Probability is 0.14, or 14%.

Venn diagrams can be useful when using the classical definition of probability, because they allow a visual representation of the number of possible outcomes.

A closely related concept is that of **mutually exclusive** or **disjoint** or **separate** events.

Definition 2 (*Mutual exclusivity*)
Two events E_1 and E_2 are called mutually exclusive if they cannot occur simultaneously.

A Venn diagram representing two mutually exclusive or disjoint events looks as follows:

Example 2
When rolling one die, if E_1 is the event that an the number 2 comes up, and if E_2 is the event that the number 3 comes up, then E_1 and E_2 are mutually exclusive.

Question 2
Why are these two events not complementary? – Can roll other #'s – like 4 or 1.

Group Exercise 2 (*7 minutes*)
Suppose that an experiment consists of tossing three fair coins. How many outcomes are in the sample space? Write down the event E_1 that corresponds to getting two of the same thing in a row (for example, one outcome in this event is (H, H, T) and another is (H, T, T); write a set containing all such outcomes). Write down the event E_2 that consists of all three coin flips having the same result. Are these events mutually exclusive?

$S = \{(H,H,H), (H,H,T), (H,T,H), (T,H,H), (H,T,T), (T,H,T), (T,T,H), (T,T,T)\}$. has 8 outcomes.

$E_1 = \{(H,H,H), (H,H,T), (T,H,H), (H,T,T), (T,T,H), (T,T,T)\}$

$E_2 = \{(H,H,H), (T,T,T)\}$

E_1 and E_2 have overlap, so not disjoint.

Group Exercise 3 (*3 minutes*)

Determine whether the pairs of events are disjoint:

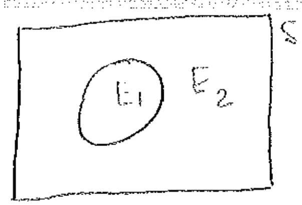
- *Randomly selecting a physician from Berkshire Medical Center and getting a surgeon; randomly selecting a physician from Berkshire Medical Center and getting a woman* Not disjoint
- *Randomly selecting a fruit fly with red eyes; randomly selecting a fruit fly with brown eyes* Disjoint
- *Receiving a phone call from a survey subject who believes there is solid evidence of global warming; receiving a phone call from a subject who is opposed to stem cell research* Not disjoint

Theorem 1

If E_1 and E_2 are two mutually exclusive events, then $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$.

Group Exercise 4 (*2 minutes*)

Use a Venn diagram to prove Theorem 1.



$$\frac{\text{Area}(E_1) + \text{Area}(E_2)}{\text{Area}(S)} = \frac{\text{Area}(S)}{\text{Area}(S)} = 1.$$

Group Exercise 5 (*2 minutes*)

What is the probability of drawing a face card or an ace from a random deck?

$$P(FC \text{ or } A) = P(FC) + P(A)$$
$$= \frac{3}{13} + \frac{1}{13} = \frac{4}{13}$$

For two events that we don't know to be mutually exclusive/disjoint, we have another rule for determining the probability of either one happening:

Theorem 2

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$.

The Venn diagram showing this rule is:

The rule is summarized as: "To find the probability that E_1 or E_2 occurs, find the number of times E_1 occurs and the number of times E_2 occurs, **adding so that each outcome is counted only once**. Then divide by the total number of outcomes in the sample space."

Recap

- Review of complements
- Mutually exclusive events
- Probability of intersection

Example 3

What is the probability of rolling one die and having the result be ≥ 4 or ≤ 5?

$$P(\geq 4 \text{ or } \leq 5) = P(\geq 4) + P(\leq 5) - P(\geq 4 \text{ and } \leq 5)$$
$$= P(\{4,5,6\}) + P(\{5,4,3,2,1\}) - P(\{4,5\})$$
$$= \frac{3}{6} + \frac{5}{6} - \frac{2}{6} = \frac{6}{6} = 1.$$

True if you think before computing.

Group Exercise 6 (*5 minutes*)

What is the probability of tossing four coins and having the result be either two heads in a row, or two tails in a row?

$S = \{(H,H,H,H), (HHHT), (HHTH), (HTHH), (THHH), (HHTT), (HTTH), (TTHH), (HTHT), (THTH), (THHT), (HTTT), (TTTH), (TTHT), (THTT), (TTTT)\}$. 16 total outcomes (i.e., $|S| = 16$).

$E_1 := \{2 \text{ heads in a row}\} = \{(H,H,H,H), (HHHT), (HHTH), (HTHH), (THHH), (HHTT), (TTHH), (TTHT)\}$

so $|E_1| = 8$, and $P(E_1) = \frac{|E_1|}{|S|} = \frac{8}{16}$

Flip outcomes

$E_2 := \{2 \text{ tails in a row}\} = \{(TTTT), (TTTH), (TTHT), (THTT), (HTTT), (TTHH), (HHTT), (HHTH)\}$

So $|E_2| = 8$, and $P(E_2) = \frac{|E_2|}{|S|} = \frac{8}{16}$

$E_1 \wedge E_2 = E_1$ and $E_2 = \{(HHTT), (TTHH)\}$, so $|E_1 \cap E_2| = 2$, and $P(E_1 \cap E_2) = \frac{|E_1 \cap E_2|}{|S|} = \frac{2}{16}$

Thus, $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2) = \frac{8}{16} + \frac{8}{16} - \frac{2}{16} = \frac{14}{16} = \frac{7}{8}.$