Lecture 9:
Lesson and Activity Packet

MATH 232: Introduction to Statistics

October 17, 2016

Homework and Announcements

- Homework 9 due in class today
- Homework 10 due on Canvas on Friday, to be submitted **in your project groups** (will be posted to Canvas later today)
- Election Prediction Project
  - Friday will be a project day
  - Please let me know before Sunday whether your group wants to present on Monday
- Quiz 5 on Wednesday (complementary and mutually exclusive events, probability of union)
- Extra credit due 11:59 p.m. on Wednesday

Questions?

Recap of today

- Probability of union
- Conditional probability

For two events that we don't know to be mutually exclusive/disjoint, we have another rule for determining the probability of either one happening:

Theorem 1

$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$.

The Venn diagram showing this rule is:

The rule is summarized as: "To find the probability that E_1 or E_2 occurs, find the number of times E_1 occurs and the number of times E_2 occurs, **adding so that each outcome is counted only once**. Then divide by the total number of outcomes in the sample space."

Example 1

What is the probability of rolling one die and having the result be ≥ 4 ***or*** ≤ 5*?*

Group Exercise 1 (*5 minutes*)

What is the probability of tossing four coins and having the result be either two heads in a row, or two tails in a row?

We now introduce **conditional probability**.
Suppose that we toss two dice, and suppose that each of the possible 36 outcomes is equally likely to occur. Suppose further that we observe the first dice is a 3 (the second one lands under the table, so we can't see it yet).

Given this information, what is the probability that the sum of the two dice equals 8?

Group Exercise 2 (*3 minutes*)

What are the outcomes that lead to a sum of 8? List them in a set called E.

Group Exercise 3 (*2 minutes*)

Since each of the outcomes that you wrote in E originally had the same probability of occurring, the outcomes still have equal probabilities; what, then, is the probability of the event E that you listed above?

If we let E and F, respectively, denote the event that the sum of the dice is 8 and the event that the first die is 3, then the probability you just computed is called the **conditional probability** that E occurs given that F has occurred, and is denoted by the expression

$$P(E|F).$$

A general formula for $P(E|F)$ that is valid for all events E and F is derived in the same manner: If the event F occurs, then in order for E to occur it is necessary that the actual occurrence be a point in both E and in F.

Definition 1

If $P(F) > 0$, then

$$P(E|F) = \frac{P(E \text{ and } F)}{P(F)}$$

Example 2

A coin is flipped twice. Assuming that all four points in the sample space are equally likely, what is the conditional probability that both flips land on heads, given that the first flip lands on heads?

Group Exercise 4

What is the conditional probability that both flips land on heads, given that at least one flip lands on heads?