L21: Mar. 10, 2017.

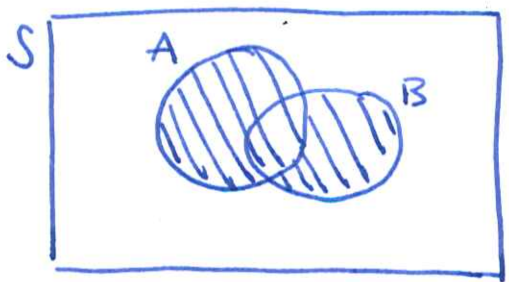
1

Housekeeping:
- $\bar{c}$ writing on Monday
- writing {disc. / summary} Monday we return
- 2 Extra credits due Weds. of break
- Essay prompts on group announcements
  Questions? — send instructor an e-mail.
- Exam 2 w/b moved fwd. by 1 week to March 31 (2nd Fri. after returning)
- 1 Extra credit assignment due after break

Rules of computing probability for "compound" events.

$$P(A \text{ or } B) = \frac{\text{Area }(A \text{ or } B)}{\text{Area }(S)} = \frac{\text{Area}(A) + \text{Area }(B) - \text{Area}(A \text{ and } B)}{\text{Area }(S)}$$

$$= \frac{\text{Area}(A)}{\text{Area}(S)} + \frac{\text{Area}(B)}{\text{Area}(S)} - \frac{\text{Area}(A \text{ and } B)}{\text{Area}(S)}$$



$$= P(A) + P(B) - P(A \text{ and } B)$$

In general, the probability of an event is: ~~the~~

$$\frac{\#\text{ of outcomes in that event}}{\#\text{ — " — the sample space}}$$

Example: $P(H) = \dfrac{\# H}{\text{total \# of ways a coin can land}} = \dfrac{1}{2}$

Example: Roll two dice; let A be the event that the two outcomes sum to 3.

$A = \{(1,2), (2,1)\}$. $|A| = 2$

$S = \{$ (on p. 31 of C.G.) $\}$. $|S| = 36$

$$P(A) = \frac{|A|}{|S|} = \frac{2}{36} = \frac{1}{18}$$

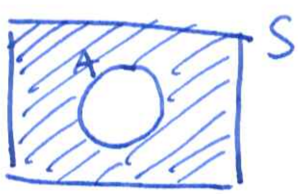
∃ (there exists)

Unfortunately, ∄ (there doesn't exist a) rule for computing P(A and B) given P(A) and P(B).

This is because we don't necessarily know by how much A & B overlap.

A rule for event complements:

$$P(A^c) = P(\text{not } A) = \frac{\text{Area}(\text{not } A)}{\text{Area}(S)} = \frac{\text{Area}(S) - \text{Area}(A)}{\text{Area}(S)}$$



$$\left(= \frac{|\text{not } A|}{|S|} = \frac{|S| - |A|}{|S|} \right)$$

$$= \frac{\text{Area}(S)}{\text{Area}(S)} - \frac{\text{Area}(A)}{\text{Area}(S)}$$

$$= 1 - P(A).$$

So, $P(A^c) = P(\text{not } A) = 1 - P(A)$.

L20, ct'd.

Special note on mutually exclusive events:

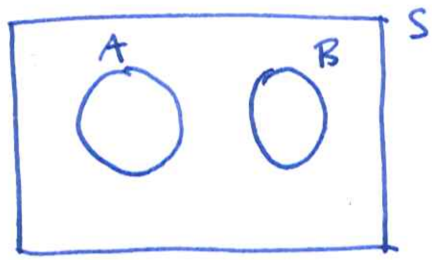
Two events are mutually exclusive when they cannot both happen — or cannot happen simultaneously.

Examples: choose a random person in a dormitory.

A : event that the person is awake

B : —— " —————— asleep

A & B are mutually exclusive.



If A & B are mutually exclusive, $P(A \text{ and } B) = 0$.

Therefore, if A & B —" ————,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= P(A) + P(B).$$

a "special rule" only for mutually exclusive events.

Cond'l Probability.

$P(A|C)$ is read "the prob. of A given C", and

is the prob. that A happens, given tht. C has happened

or ... ——— " ————— , <u>if</u> C happens.

Example. $P(\text{dark} \mid \text{midnight}) = 1$ (in North Adams)

$P(\text{midnight} \mid \text{dark}) = \,? \neq 1$; much smaller!

In general,
$$P(A|B) = \frac{\text{Area}(A \text{ and } B)}{\text{Area}(B)} \cdot \overbrace{\frac{1/\text{Area}(S)}{1/\text{Area}(S)}}^{1}$$

$$= \frac{\left(\text{Area}(A \text{ and } B)/\text{Area}(S)\right)}{\left(\text{Area}(B)/\text{Area}(S)\right)}$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$