L28: Wednesday, April 5. 1

Housekeeping: · Written HW due today

· —"— Friday (check Canvas).

· Week 8 {summary, discussion post} due Monday (last ones?)

· Book project: essay draft due Apr. 19 on Canvas 11:59 p.m.
(& bring hard copy to class Apr. 21 for peer review)

Last time: What are discrete random variables? & Probability Distribut'n Fns./Tables

QUESTIONS?

This time: Mean & expected values of discrete random variables

Std. deviation of a d. r.v.

 Mean of a r.v. = Expected value = long-term avg.

To compute the mean of a discrete random variable, multiply each value of the random variable by the probability of that value, and sum the results.

Example: $X :=$ # of days in a given week that Nancy attends class

x	$P(X=x)$	$x \cdot P(X=x)$
0	0.01	0
1	0.04	0.04
2	0.15	0.30
3	0.80	2.40

$\boxed{\mu(X) = 2.74}$

MEAN: $\boxed{\mu(X) = \underline{0 \cdot 0.01} + \underline{1 \cdot 0.04} + \underline{2 \cdot 0.15} + \underline{3 \cdot 0.80}} =$

$= 0 + 0.04 + 0.30 + 2.4$

$= 0 + 0.04 + 0.30 + 2.4$

$= 0.34 + 2.4$

$\boxed{\mu(X) = 2.74}$

Note: The mean $\mu(X)$ for a discrete r.v. is also called the expected value of that r.v. — you can think of it as a "long-term average".

Note: The "mean of a random variable" is a DIFFERENT CONCEPT than the "mean of a data set".

Ex. 4.3 p. 244

Women's soccer team plays zero, one, or two days per week. Let $X :=$ # of days the soccer team plays.

PDF:

x	$P(X=x)$
0	0.2
1	0.5
2	0.3

① Check tht. it's really a PDF

② Compute the mean/expected value / l.t. avg.

②
$$\mu(X) = 0(0.2) + 1(0.5) + 2(0.3)$$
$$= 0 + 0.5 + 0.6$$
$$= 1.1$$

①
$$\left.\begin{array}{l} 0 \le 0.2 \le 1 \\ 0 \le 0.5 \le 1 \\ 0 \le 0.3 \le 1 \end{array}\right\} \checkmark$$

$\checkmark$ $0.2 + 0.5 + 0.3 = 1$

0.5 1

Ex. 4.4 p. 244

$X :=$ the # of times per week a newborn wakes his/her mother after midnight.

x	$P(X=x)$	$x \cdot P(X=x)$
0	2/50	
1	11/50	
2	23/50	
3	9/50	
4	4/50	
5	1/50	

FOUR

Compute $\mu(X)$.

$$\mu(X) = 2.1$$
$$= 0 \cdot \frac{2}{50} + \frac{1 \cdot 11}{50} + \frac{2 \cdot 23}{50} + \frac{3 \cdot 9}{50} + \frac{4 \cdot 4}{50} + \frac{5 \cdot 1}{50}$$

① (a) Are all probabilities btwn. 0 & 1 ?

$0 \le 0.2 \le 1$ ✓ — YES.

$0 \le 0.5 \le 1$ ✓

$0 \le 0.3 \le 1$ ✓

(b) Do the probabilities in the table sum to 1?

i.e., $0.2 + 0.5 + 0.3 \overset{\checkmark ?}{=} 1$ — YES.

0.5 1

So, YES - a PDF.

②

x	$P(x)$	$x \cdot P(x)$
0	0.2	$0(0.2) = 0$
1	0.5	$1(0.5) = 0.5$
2	0.3	$2(0.3) = 0.6$ +

$$\mu(X) = 0 + 0.5 + 0.6 = 1.1$$

Over the long term, expect that on a given week, the women soccer team plays 1.1 times.

Like data sets, probability distributions have <u>standard deviation</u>. They are computed similarly:

1. Find the mean $\mu(X)$
2. For each value x, compute $(x-\mu)^2 \cdot P(X=x)$
3. Sum up those values from (2)
4. Take the square root of the value from (3).

If there are n many values that the r.v. takes on — say, $x_1, x_2, \dots, x_n$ — and if μ is the mean of X, then

EQUIVALENT!

$$\sigma(X) = \sqrt{\sum_{i=1}^{n} (P(X=x_i)) \cdot (x_i-\mu)^2}$$

$a^c \cdot b^c = (a \cdot b)^c$

$(x-\mu)^2 \overset{?}{=} (\mu-x)^2$

$(x-\mu)^2 = ((-1)(\mu-x))^2$

$(x-\mu)^2 = (-1)^2(\mu-x)^2$

<u>Example</u>. X is as in Ex. 4.4, p. 244. $\mu(x) = 2.1$

$(x-\mu)^2 = (\mu-x)^2$

x	$P(X=x)$	$(2.1-x)^2 \cdot P(X=x)$
0	2/50	$(2.1-0)^2 \cdot \frac{2}{50} = \frac{(2.1)^2 \cdot 2}{50}$
1	11/50	$(2.1-1)^2 \cdot 11/50 = \frac{(1.1)^2 \cdot 11}{50}$
2	23/50	$(2.1-2)^2 \cdot 23/50 = \frac{(0.1)^2 \cdot 23}{50}$
3	9/50	$(2.1-3)^2 \cdot 9/50 = \frac{(-0.9)^2 \cdot 9}{50} = \frac{(0.9)^2 \cdot 9}{50}$
4	4/50	$(2.1-4)^2 \cdot 4/50 = \frac{(-1.9)^2 \cdot 4}{50}$
5	1/50	$(2.1-5)^2 \cdot 1/50 = \frac{(-2.9)^2 \cdot 1}{50}$

Ex., ct'd.

$$\sigma(X) = \sqrt{\frac{2.1^2 \cdot 2 + 1.1^2 \cdot 11 + 0.1^2 \cdot 23 + 0.9^2 \cdot 9 + 1.9^2 \cdot 2 + 2.9^2 \cdot 1}{50}}$$

$$\approx \boxed{0.95}$$

Ex. $X :=$ # of days/wk. the women's soccer team plays.

x	$P(X=x)$	$x \cdot P(x)$	$(\mu - x)^2 \cdot P(x)$
0	0.2	0	$(1.1-0)^2 \cdot 0.2 = 0.242$
1	0.5	0.5	$(1.1-1)^2 \cdot 0.5 = 0.005$
2	0.3	0.6	$(1.1-2)^2 \cdot 0.3 = 0.243$
		+ $\mu(X) = 1.1$	+ $\sigma^2(X) = 0.49$

$$\sigma(X) = \sqrt{\cancel{\sigma^2(X)}\ 0.49} = \underline{0.7}.$$