L37 (Final class!) : May 1, 2017

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Housekeeping:

- Poisson distr. Extra credit due May 5 before your exam (on Canvas or hard copy)
- Final essay due May 5, 11:59 p.m. on Canvas
- "Final" exam/Exam 3 May 5 $\begin{cases} 10{:}30 \text{ am} \\ 1{:}00 \text{ pm} \end{cases}$
- A21 (?) due today.

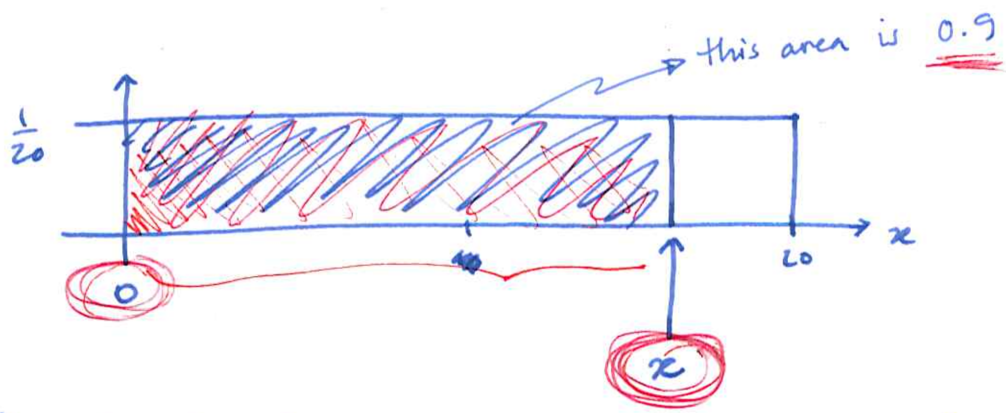
Percentiles.

What if we were told $X \sim \text{unif}(0, 20)$ and asked to find the 90^{th} percentile of data that fit that distribution?

That is: find the value of x, such that

$$P(X < x) = 90\% = 0.9.$$

Since the range of X is btwn. 0 and 20, $P(X < x)$, can be written $P(0 < X < x)$. Find x s.t. :



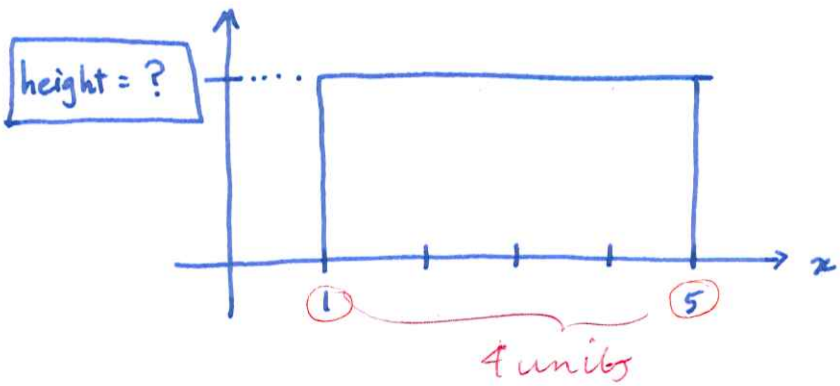
Height of rectangle is always $\frac{1}{20}$. In terms of x, the base is $x - 0 = x$ units long, so the area of the rectangle is $\frac{x}{20} = x \cdot \left(\frac{1}{20}\right)$.

width · height

If we want $\boxed{\frac{x}{20} = 0.9}$, then $x = 0.9 \cdot 20 = 18$.

So the 90^{th} percentile is 18.

Example. If $X \sim \text{unif}(1, 5)$ then what is the 80^{th} pctl?



First: Find height of rectangle (need t/know).

Use the fact that total area must be 1 !

base = $5 - 1 = 4$ units long

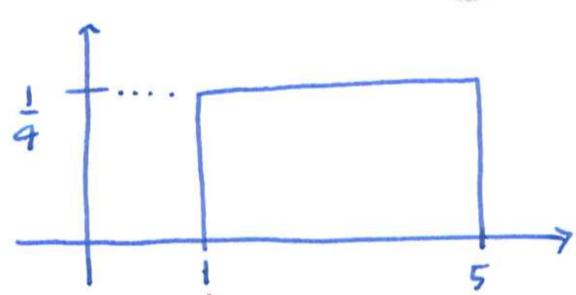
Know area = base $\cdot$ height, so

$$\text{height} = \frac{\text{area}}{\text{base}},$$

or for our case,

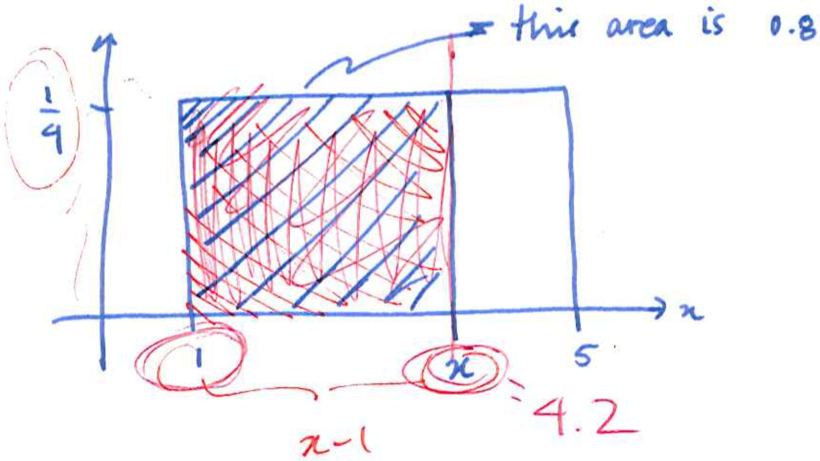
$$h = \frac{1}{4}.$$

So have



Example, ct'd.

Want t/find x s.t. $P(1 < X < x) = 0.8$, i.e.,



Well, for the shaded rectangle, base $= x-1$.

Want ~~base height area~~ area to be 0.8, so use these in the formula:

$$\text{base} \cdot \text{height} = \text{area}$$

$$\text{base} = \frac{\text{area}}{\text{height}}$$

$$x - 1 = \frac{0.8}{1/4}$$

$$\frac{0.8}{1/4} = 4(0.8)$$

$$x - 1 = 4(0.8)$$

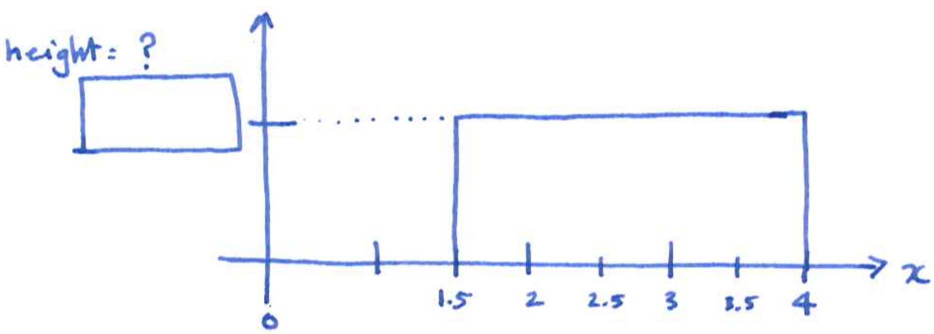
$$x - 1 = 3.2$$

$$x = 4.2$$

So the 80^{th} percentile is 4.2.

Question (in groups):

If $X \sim \text{unif}(1.5, 4)$, then what is the 80^{th} percentile?



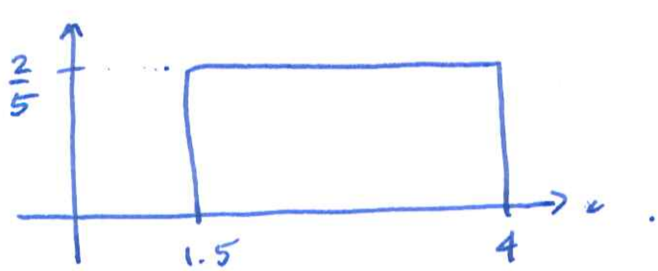
First: What's the height of rectangle? (Have to know to find percentile.)

Well, need TOTAL area equal to 1.

Know total base $= 4 - 1.5 = 2.5 = \frac{5}{2}$,

and if $b \cdot h = 1$, that means $h = \frac{1}{b} = \frac{1}{5/2} = \frac{2}{5}$.

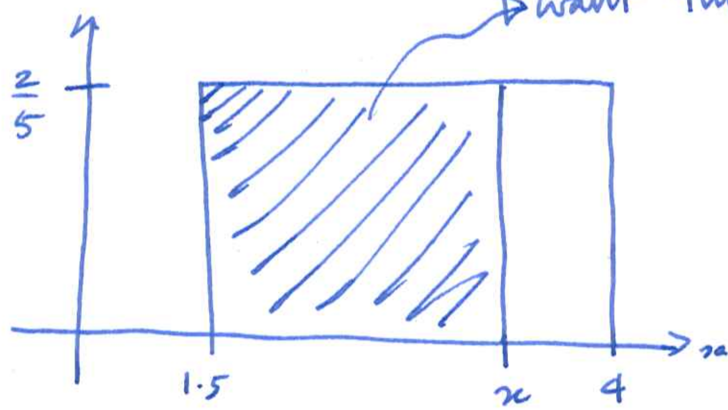
So now we have



So we want to find x such that $P(1.5 < X < x) = 0.8$.

That is:

want this area to be 1.



Height is still $\frac{2}{5}$.

Base is $x - 1.5 = x - \frac{3}{2} = \frac{2x-3}{2}$.

Want $b \cdot h = 0.8$, i.e., $b = \frac{0.8}{h} = \frac{8}{10\,h}$,

So $\frac{2x-3}{2} = \frac{8}{10 \cdot \left(\frac{2}{5}\right)}$,

i.e., $\frac{2x-3}{2} = \frac{8}{2 \cdot 2}$

i.e., $\frac{2x-3}{2} = \frac{4}{2}$

i.e., $2x - 3 = 4$

i.e., $2x = 7$, i.e., $x = \frac{7}{2} = 3.5$.

L30? ct'd.

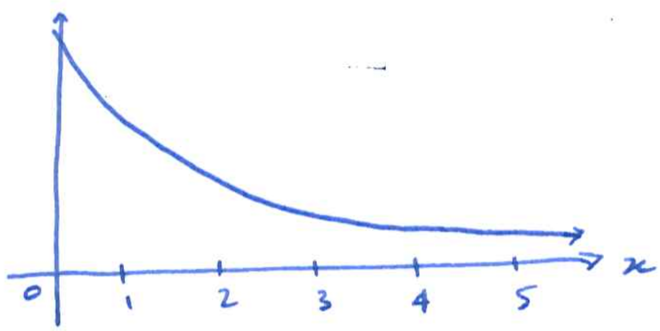
For a UNIFORM r.v., $X \sim unif(a,b)$

$$\mu[X] = \frac{b-a}{2}$$ (it's the 50^{th} percentile!) (the "CENTER OF THE DISTRIBUT'N")

$$\sigma^2[X] = \frac{(b-a)^2}{12}, \text{ so } \sigma[X] = \frac{b-a}{2\sqrt{3}}.$$

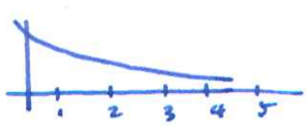
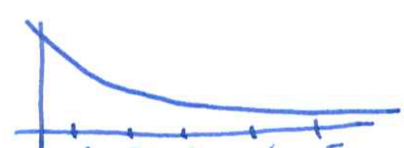
IN GROUPS...

Suppose the following is the PDF for an r.v. X:



"EXPONENTIAL DISTRIBUT'N"

Shade the areas corresponding to:

- $P(X < 4)$ 
- $P(X > 2)$ 
- $P(1 < X < 3)$ 
- $P(X < 1 \text{ or } X > 3)$ 

HW 21: ch. 5, № 74 (a)–(c), (f)–(h)

Births are assumed t/b uniformly distributed btwn. weeks 1 and 52 of the year.

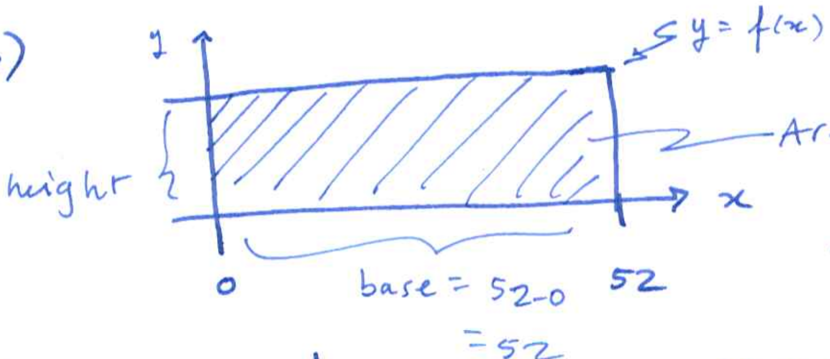
Let $X :=$ ~~[illegible]~~ the week that a random baby is born.

Then X takes values $1, 2, 3, \ldots, 52$ if weeks cannot have frac'l parts – then a discrete r.v..

Or, if X is measured in "time" – then units can be more precise, i.e., X is a cts. r.v. that takes any value in the interval $[0, 52]$.

(a) $X \sim \text{unif}(0, 52)$

(b)



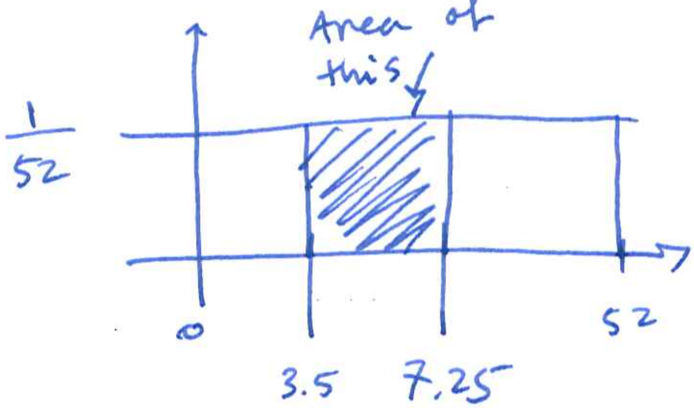
Area = base · height

height $= \dfrac{\text{Area}}{\text{base}} = \dfrac{1}{52}$

(c) $f(x) = \dfrac{1}{52}$ (Probability density fn.)

(f) $P(3.5 < x < 7.25)$ = Area ~~of~~ under the PDF curve, over the interval $[3.5, 7.25]$

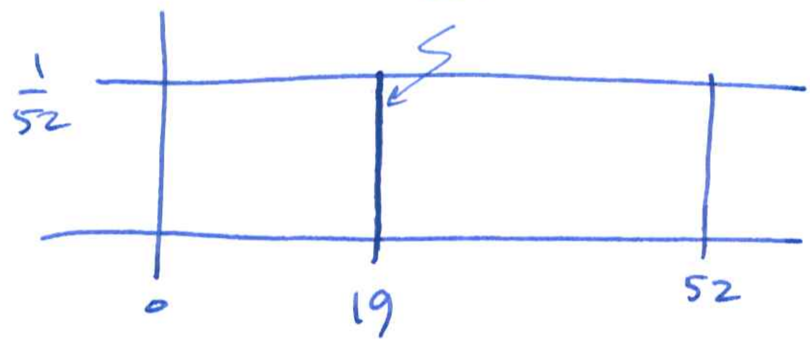
=



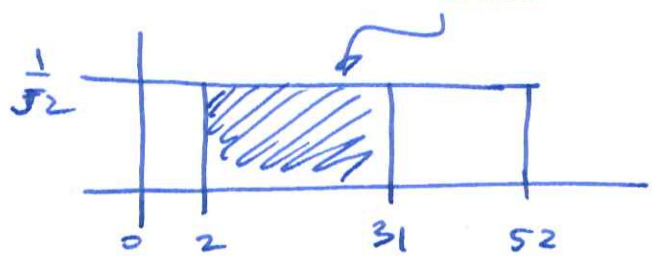
Area = base · height

$= (7.25 - 3.5) \cdot \left(\frac{1}{52}\right)$

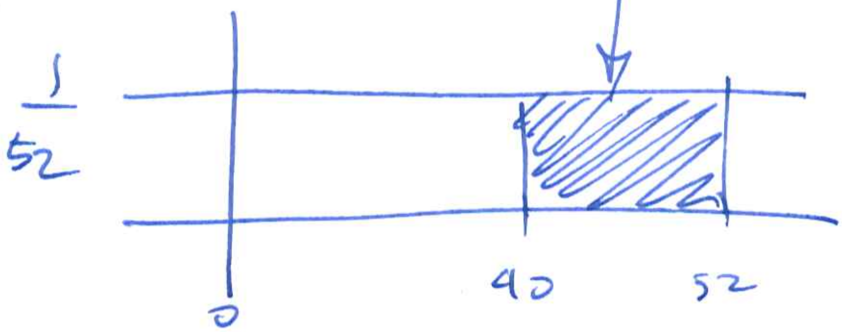
(f) $P(X = 19)$ = this area $= 0$.



(g) $P(2 < X < 31)$ = this area $= (31-2) \cdot \left(\frac{1}{52}\right) = \frac{29}{52}$



(h) $P(X > 40) = \text{area} = \frac{1}{52} \cdot (52 - 40) = \frac{12}{52}$



$= \frac{6}{26}$

$= \frac{3}{13}$