MATH 261 · Linear Algebra
Spring 2017

Written Homework 1: Due in class January 31

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. Consider the following system of linear equations:

$$\begin{cases} 2x_1 - 2x_2 + 2x_3 - 2x_5 = 16 \\ x_1 + x_2 + 5x_3 + 9x_5 = 8 \\ -x_1 - 3x_3 + x_4 + 2x_5 = -1 \\ x_1 + 3x_3 + 4x_5 = 8. \end{cases}$$

(a) Write down the augmented matrix corresponding to this system.

(b) Use the row reduction algorithm on this matrix to obtain a row-equivalent matrix in **reduced row echelon form**.

(c) Using part (b), find all solutions to the original linear system. Describe the solution set in parametric form, as shown in class.

Problem 2. Not all algebraic systems are as intuitive as the real numbers. For example, if $A := \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$, then $A^2 := AA = 0$, where 0 denotes the 2×2 matrix of all zeros. In fact, there are infinitely many 2×2 matrices A satisfying $A^2 = 0$. Systematically determine all 2×2 matrices $A := \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ which satisfy $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. [Hint: Split your analysis into two cases: one where $b = 0$, and another assuming that $b \neq 0$.]

Problem 3. You are already familiar with the **vector space** $\mathbb{R}^n$ consisting of all $n \times 1$ matrices with real entries. But in this course, we will eventually discuss other vector spaces than this one. For example, the set of all continuous functions defined on a given real interval is a vector space. In this problem, we will analyze the following "dot product" on the space of all functions that are continuous on $[0, 1]$:

$$f \cdot g := \int_0^1 f(x)g(x)\,\mathrm{d}x.$$

Recall that two vectors in a vector space are **orthogonal** if their dot product is zero. [For example, $\cos(\pi x)$ is orthogonal to $\sin(\pi x)$ in this vector space.] By computing the above dot product for each of the six possible pairs[2], determine which of the following functions are orthogonal to each other:

(a) $a(x) := 1$ for $0 \leq x \leq 1$

(b) $b(x) := x$ for $0 \leq x \leq 1$

(c) $c(x) := x - \frac{1}{2}$ for $0 \leq x \leq 1$

(d) $d(x) := 6x^2 - 6x + 1$ for $0 \leq x \leq 1$

Note! In signal processing, signals represented by orthogonal functions can be easily separated by a filter. This is how a mobile phone network can allow multiple users on the same frequency.

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[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

[2] That is, simply compute the integrals defined by $a \cdot b$, $a \cdot c$, $a \cdot d$, $b \cdot c$, $b \cdot d$, and $c \cdot d$.