MATH 261 · Linear Algebra
Spring 2017

## Written Homework 2: Due in class February 7

### Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

**Problem 1.** Recall from class that a "diagonal" matrix is one that has nonzero entries *only* on the main diagonal. [That is, if $A \in \mathbb{R}^{n\times n}$ is a matrix whose entries are denoted $a_{ij}$, then $a_{11}$, $a_{22}$, $\ldots$, $a_{nn}$ are the only entries that are allowed to be nonzero.] Prove that for two arbitrary diagonal matrices, their product is also diagonal.

**Problem 2.** If $A$ and $B$ are diagonal matrices, is their multiplication commutative? [That is, does $AB = BA$?]

**Problem 3.** A *symmetric* matrix is one that equals its transpose. That is, $A$ is symmetric if and only if $A^\top = A$. A matrix is *skew symmetric* if and only if $A^\top = -A$. Prove that for **any** matrix $A \in \mathbb{R}^{n\times n}$, $A + A^\top$ is symmetric, and $A - A^\top$ is skew-symmetric.

**Problem 4.** Find an equation relating $a$, $b$, and $c$ so that the linear system

$$\begin{cases} x + 2y - 3z &= a \\ 2x + 3y + 3z &= b \\ 5x + 9y - 6z &= c \end{cases}$$

is *consistent*—that is, find an equation relating $a$, $b$, and $c$ so that there exists at least one solution to the system.

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`