MATH 261 · Linear Algebra
Spring 2017

Written Homework 3: Due in class February 28

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. Let $A \in \mathbb{R}^{n\times n}$. Show that A is invertible if and only if the only solution to $A\vec{x} = \vec{0}$ is the trivial solution $\vec{x} = \vec{0}$. Remember that there are two directions to prove.

Problem 2. Suppose $\theta \in [0, 2\pi]$ is fixed. Show that the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is invertible, and compute its inverse.

Problem 3. Show that if A is nonsingular and symmetric, then A^{-1} is also symmetric. [Hint: Use the fact that for any two invertible matrices, $(AB)^{-1} = B^{-1}A^{-1}$.]

Problem 4. Show that if A is singular and $A\vec{x} = \vec{b}$ has a nontrivial solution, then it has infinitely many nontrivial solutions.

Problem 5. Find all values of a for which the inverse of $A := \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 2 & a \end{bmatrix}$ exists. What is A^{-1}?

[1]See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`