L1: Jan. 19, 2017. 1

Announcements / Assignments

- Please read syllabus
- No written or online homework due Tuesday ~~this w~~
- Download textbook (link in syllabus), but when the online HW platform is operable, I'll send you a new version with the correct links.

Questions?

Today

- What is Lin. Alg.?
- Matrices + systems of eq'ns.

Linear algebra is the study of <u>linear functions</u>, <u>vectors</u>, and <u>matrices</u>.

<u>Vectors</u> are objects that can be added & multiplied by scalars, e.g.:

- <u>Numbers</u>: 3 & 5 are #s, and so is 3+5.
- <u>Vectors in $\mathbb{R}^3$</u>: $\begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\2\\1\end{pmatrix}$ by standard rules of vector add'n.
- <u>Polynomials</u>: if $p(x) := 1 + x - 2x^2 + 3x^3$, and if $q(x) := x + 3x^2 - 3x^3 + x^4$, then $p(x) + q(x) = 1 + 2x + x^2 + x^4$, which is also a polynomial. Also, $3p(x) = 3 + 3x - 6x^2 + 9x^3$ ← STILL A POLYN.
- <u>Convergent power series</u>: if $f(x) := 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots$, and $g(x) := 1 - x + \frac{1}{2!}x^2 - \frac{1}{3!}x^3 + \cdots$, then (after proving both series converge), $f(x) + g(x) := 2 + x^2 + \frac{2}{4!}x^4 + \frac{2}{6!}x^6 + \cdots$, which is also a convergent power series.

NOT JUST "STACKS OF NUMBERS"!

There are more precise rules that vectors & their operations (add'n & scalar mult.) need to obey — but, for now, think of vectors as "things that can be added to each other and multiplied by scalars"...

Linear functions are functions of vectors that "respect" vector addition & scalar multiplication:

$$\left.\begin{array}{l} \cdot\ \ f(u+v) = f(u) + f(v) \\ \cdot\ \ f(cu) = c\,f(u) \end{array}\right\}$$

for all $u, v \in \mathrm{dom}(f)$ and all $c \in \mathbb{R}$ (or in whatever other field of scalars)

Examples.

- $f(x) = 10x$. $\mathrm{dom}(f) = \mathbb{R}$.

Well, $f(x+y) = 10(x+y) = 10x + 10y = f(x) + f(y)$

$f(cx) = 10(cx) = (10c)x = (c\cdot 10)x = c(10x) = cf(x)$

- $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x+y \\ 0 \\ 0 \end{pmatrix}$. $f: \mathbb{R}^3 \to \mathbb{R}^3$.

Well, $f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = f\left(\begin{pmatrix} x+a \\ y+b \\ z+c \end{pmatrix}\right) = \begin{pmatrix} x+a+y+b \\ 0 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} x+y+a+b \\ 0 \\ 0 \end{pmatrix}$

$= \begin{pmatrix} x+y \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} a+b \\ 0 \\ 0 \end{pmatrix}$

$= f\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) + f\left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}\right)$

$f\left(c\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = f\left(\begin{pmatrix} cx \\ cy \\ cz \end{pmatrix}\right) = \begin{pmatrix} cx+cy \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c(x+y) \\ 0 \\ 0 \end{pmatrix}$

$= c\begin{pmatrix} x+y \\ 0 \\ 0 \end{pmatrix}$

$= c\,f\begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

- Derivatives:

$$\frac{d}{dx}\left[f(x)+g(x)\right] = \frac{d}{dx}\left[f(x)\right] + \frac{d}{dx}\left[g(x)\right]$$

$$\frac{d}{dx}\left[c f(x)\right] = c\frac{d}{dx}\left[f(x)\right].$$

Examples of non-linear fns:

- $f(x) := x^2$.

$$f(x+y) = (x+y)^2 = x^2+2xy+y^2 \neq x^2+y^2 = f(x)+f(y).$$

Also, $f(cx) = (cx)^2 = c^2x^2 \neq cx^2 = cf(x)$.

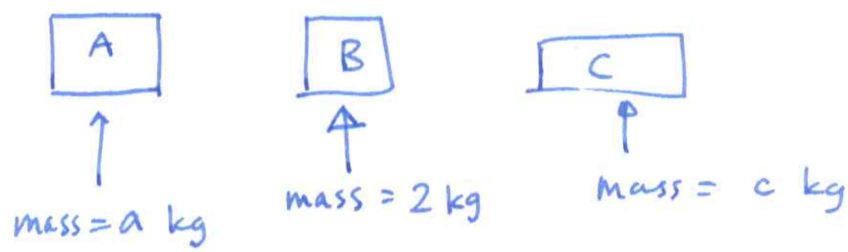
- Prove that $f(x) := e^x$ is non linear.

Matrices.

Broadly, matrices are the result of organizing information related to linear functions.

Canonical example: systems of linear equations.

Example. Suppose we have 3 objects:



∴ using a meter stick produces 2 configurations that balance:

Balance means the sum of the moments on the left is the same as the sum of the moments on the right, and the moment = (mass)(dist. from balance pt.)

FIRST BALANCE: $40a + 15c = 50(2)$

SECOND BALANCE: $25c = 25(2) + 50a$

$2a - c = -2$

So, the system of linear eq'ns is:

$$\begin{cases} 40a + 15c = \cancel{\$}100 \\ 2a - c = -2 \end{cases}$$

Can solve for the unknown masses:

First, scale eq'n (1):

$$\begin{cases} 8a + 3c = \cancel{\$} 20 \\ 2a - c = -2 \end{cases}.$$

Observation: · Adding the same thing to both sides of an eq'n doesn't affect that eq'n's truth or falsehood.

Observation: Eq'n (2) is true, because it came from physics. So $2a-c$ is the same thing as -2.

Observation: Eq'n (1) is true.

$$8a + 3c + (2a-c) = 20 + \underbrace{(2a-c)}_{=-2}$$

$$8a + 3c + (2a-c) = 20 - 2$$

$$10a + 2c = 18.$$

$$\begin{cases} 8a + 3c = 20 \\ 2a - c = -2 \end{cases}.$$

$$\text{Eq}(1) - 4\cdot\text{Eq}(2): \quad 8a + 3c - 4(2a-c) = 20 - 4(-2)$$

$$8a + 3c - 8a + 4c = 20 + 8$$

$$0a + 7c = 28$$

$$7c = 28$$

$$\text{Eq}(3): \quad c = 4.$$

$$\text{Eq}(2) + \text{Eq}(3): \quad 2a - c + c = -2 + 4$$

$$2a = 2$$

$$a = 1$$

$\begin{cases} 8a + 3c = 20 \\ 2a - c = -2 \end{cases}$ is equivalent to the augmented system $\left(\begin{array}{cc|c} 8 & 3 & 20 \\ 2 & -1 & -2 \end{array}\right)$

$$\left(\begin{array}{cc|c} 8 & 3 & 20 \\ 2 & -1 & -2 \end{array}\right) \longrightarrow \begin{array}{c} \text{EQ1} \\ -4\text{EQ2}+\text{EQ1} \end{array}\left(\begin{array}{cc|c} 8 & 3 & 20 \\ 0 & 7 & 28 \end{array}\right) \rightarrow \begin{array}{c} \text{E1} \\ \frac{\text{E2}}{7} \end{array}\left(\begin{array}{cc|c} 8 & 3 & 20 \\ 0 & 1 & 4 \end{array}\right) \rightarrow$$

$$\rightarrow \begin{array}{c} \text{E1}-3\text{E2} \\ \text{E2} \end{array}\left(\begin{array}{cc|c} 8 & 0 & 8 \\ 0 & 1 & 4 \end{array}\right) \rightarrow \begin{array}{c} \text{E1}/8 \\ \text{E2} \end{array}\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 4 \end{array}\right)$$

$$\left.\begin{array}{l} a\cdot 1 + c\cdot 0 = 1 \\ a\cdot 0 + c\cdot 1 = 4 \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} a = 1 \\ c = 4 \end{array}}$$