L2: Jan. 24, 2017.

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Housekeeping.

- WeBWork server space ~~are~~ accessible!

  Please log in + play around — ~~also~~ today's homework will be assigned using WW.

- Unfortunately, the updated book (with web links to our WW course) isn't ready yet... so for the next few assignments, I'll tell you the names of the problem sets.

- HIDDEN FIGURES: 6:40 & 9:20 p.m. @ Movieplex, free w. UCLA student ID ($5 others), lineup @ 6 p.m.

Deliverables.

- WeBWork — Background
  — What Is Linear Algebra

  } due Thursday, 2 p.m.

- Next week, Tuesday (Jan. 31) — written assignment due in class (see Canvas)

Last time: Basics (what is lin. alg., linearity, etc.)
Matrices as coefficient-holders. QUEST'NS?

Today: Matrix operations — add'n, mult. by scalars, and matrix multiplicat'n. Possibly row reduc'n.

Matrices.

Usually enclosed with square brackets or parentheses:

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}, \quad \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \qquad \text{(these are equivalent).}$$

General form:

An "m-by-n" matrix

OR

A matrix in $\mathbb{R}^{m \times n}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & & a_{2n} \\ \vdots & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Means m many rows & n many columns.

a_{ij} refers to the entry of the matrix in th i^{th} row and j^{th} column.

Example: If $A := \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$, then

$a_{11} = 2$ $\quad$ $a_{12} = 4$

$a_{21} = 6$ $\quad$ $a_{22} = 8$.

$A \in \mathbb{R}^{2\times 2}$,

A is 2×2.

Vectors are kinds of matrices!

$$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$, a "column vector"

$$\begin{bmatrix} 2 & 1 & 8 \end{bmatrix} \in \mathbb{R}^{1\times 3}$$, a "row vector"

Add'n

- Can add two matrices only when they have the same dimensions!

e.g.,

$$\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2+1 & 4+3 \\ 6+5 & 8+7 \end{pmatrix} = \begin{pmatrix} 3 & 7 \\ 11 & 15 \end{pmatrix}$$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

but $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is nonsense.

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 7 & 8 \end{bmatrix}$$

Scalar mult.

Can mult. matrices by scalars: $3\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 3\cdot 2 & 3\cdot 4 \\ 3\cdot 6 & 3\cdot 8 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 18 & 24 \end{pmatrix}$

Matrices.

Try in groups:

$$A := \begin{array}{c} \begin{array}{ccc} & & a_{13} \end{array} \\ \begin{bmatrix} 1 & 0 & 4 \\ 5 & 2 & 0 \\ 8 & 6 & 1 \end{bmatrix} \end{array}, \quad B := \begin{bmatrix} 11 & 3 & 0 \\ 7 & 4 & 1 \\ 1 & 2 & 9 \end{bmatrix}$$

$$\vec{u} := \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \quad \vec{v} := \begin{bmatrix} 2 & 8 & 6 \end{bmatrix}, \quad \vec{w} := \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$$

- What is a_{31} ? 8
- What is a_{13} ? 4
- What is $A + B$? $\begin{bmatrix} 12 & 3 & 4 \\ 12 & 6 & 1 \\ 9 & 8 & 10 \end{bmatrix}$
- — " — $\vec{u} + \vec{v}$? — NONSENSE!
- — " — $\vec{u} + \vec{w}$? $\begin{bmatrix} 5 \\ 9 \\ 11 \end{bmatrix}$
- — " — $2 \cdot A$? $\longrightarrow \begin{bmatrix} 2 & 0 & 8 \\ 10 & 4 & 0 \\ 16 & 12 & 2 \end{bmatrix}$
- — " — $3 \cdot \vec{u}$? $\begin{bmatrix} 9 \\ 3 \\ 15 \end{bmatrix}$

Matrix multiplication.

- Can multiply two matrices <u>only</u> if their "inner" dimensions match, e.g.:

<u>can</u> multiply: $\overset{1^{st}\text{ matrix}}{3 \times 3}$ by $\overset{2^{nd}\text{ matrix}}{3 \times 1}$

=

2×2 by 2×5

=

<u>cannot</u> multiply: 3×3 by 1×3

$\neq$

2×2 by 3×1

$\neq$

- When you multiply two matrices, the resulting product has the "outer" dimensions, e.g.:

The product of a (2×2) with a (2×1)

=

is a 2×1

e.g.: 3×4 with 4×7 is ...

3×7

1×3 with 3×1 is 1×1 (a scalar!)

Wait a minute... $\underbrace{(1 \times 3)}_{\text{row vector}} \otimes_M \underbrace{(3 \times 1)}_{\text{column vector}}$ gives a scalar?

The "matrix product" of those particular 2 vectors is a scalar.

Recall: DOT PRODUCT.

e.g., $\langle 3, 1, 0 \rangle \cdot \langle 1, 5, 8 \rangle = \underline{3(1) + 1(5) + 0(8) = 8}$

$\langle 2, 0, 7 \rangle \cdot \langle 3, 3, 2 \rangle = \underline{20}$

In fact,

$$\begin{bmatrix} 3 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} = 8$$

also!

$$\begin{bmatrix} 2 & 0 & 7 \end{bmatrix} \times \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = 20$$

So, we know when we're allowed to multiply two matrices, and we know the dimension of the product.

We also know that for matrices of the shapes $1 \times n$ and $n \times 1$, their product is the same as the dot product.

How to actually compute matrix products??

Let $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$.

Then $A \times B$ exists ($A \cdot B$, AB work too), because

$$(m \times n) \times (n \times p) .$$

$$=$$

The product, call it C, is in $\mathbb{R}^{m \times p}$.

For all $i \in \mathbb{N} \cap [1, m]$ and all $j \in \mathbb{N} \cap [1, p]$,

$$C_{ij} = \underbrace{\left(i^{\underline{th}} \text{ row of } A\right)}_{\text{is a vector— } (1 \times n)} \cdot \underbrace{\left(j^{\underline{th}} \text{ col. of } B\right)}_{(n \times 1) \text{ vector}}$$

<u>Example.</u> Let $A := \begin{bmatrix} 9 & 3 & 6 \\ 5 & 2 & 0 \end{bmatrix} \in \mathbb{R}^{2\times 3}$

$$B := \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \in \mathbb{R}^{3\times 1}$$

Can compute the product : $(2 \times 3) \times (3 \times 1)$

$=$

$C := AB$ implies $C \in \mathbb{R}^{2\times 1}$

$$C = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} = \begin{bmatrix} 21 \\ 5 \end{bmatrix}$$

$c_{11} = (1^{\text{st}}$ row of $A) \cdot (1^{\text{st}}$ col. of $B)$

$$= \begin{bmatrix} 9 & 3 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 9(1) + 3(0) + 6(2)$$

$$= 9 + 12 = 21$$

$c_{21} = (2^{\text{nd}}$ row of $A) \cdot (1^{\text{st}}$ col of $B)$

$$= \begin{bmatrix} 5 & 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 5(1) + 2(0) + 0(2) = 5$$

Let $A := \begin{bmatrix} 6 & 0 & 5 & 2 \\ 9 & \cancel{4}\,7 & 4 & 3 \end{bmatrix} \in \mathbb{R}^{2\times 4}$

$B := \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ 3 & 0 \\ 6 & 0 \end{bmatrix} \in \mathbb{R}^{4\times 2}$

So if $C := AB$, then $C \in \mathbb{R}^{2\times 2}$.

Let $C := \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 33 & 24 \\ 53 & \cancel{34}\ 36 \end{bmatrix}$

$c_{12} = (1^{\text{st}} \text{ row of } A) \cdot (2^{\text{nd}} \text{ col. of } B)$

$= \begin{bmatrix} 6 & 0 & 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 24$

Matrix transposes.

Example: If $A = \begin{bmatrix} 6 & 0 & 5 & 2 \\ 9 & 7 & 4 & 3 \end{bmatrix}$,

then $A^T = \begin{bmatrix} 6 & 9 \\ 0 & 7 \\ 5 & 4 \\ 2 & 3 \end{bmatrix}$. $(A^T)^T = A$.

In general, if $A \in \mathbb{R}^{n \times m}$, then $A^T \in \mathbb{R}^{m \times n}$,

and the i^{th} row of A^T is the same as the i^{th} col. of A, where $i \in \mathbb{N} \cap [1, m]$.