

L2: Jan. 24, 2017.

Housekeeping.

- WeBWork server space ~~is~~ accessible!

Please log in + play around - ~~also~~ today's homework will be assigned using ww.

- Unfortunately, the updated book (with web links to our ww course) isn't ready yet... so for the next few assignments, I'll tell you the names of the problem sets.

- HIDDEN FIGURES: 6:40 \approx 9:20 p.m. @ Movieplex, free w. MCLA Student ID (\$5 others), lineup @ 6 p.m.

Deliverables.

- WeBWork - Background
- What Is Linear Algebra } due Thursday, 2 p.m.

- Next week, Tuesday (Jan. 31) - written assignment due in class (see Canvas)

Last time: Basics (what is lin. alg., linearity, etc.)
Matrices as coefficient-holders. QUEST'NS?

Today: Matrix operations - add'n, mult. by scalars, and matrix multiplication. Possibly row reduc'n.

Matrices.

Usually enclosed with square brackets or parentheses:

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}, \quad \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} \quad (\text{these are equivalent}).$$

General form:

An "m-by-n" matrix
OR
A matrix in $\mathbb{R}^{m \times n}$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & \dots & a_{2n} \\ \vdots & a_{32} & a_{33} & \dots & a_{3n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mn} \end{bmatrix}$$

Means m many rows $\dot{\vdots}$ n many columns.

~~Matrix~~

a_{ij} refers to the entry of the matrix in
th i^{th} row and j^{th} column.

Example: If $A := \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$, then

$$a_{11} = 2$$

$$a_{12} = 4$$

$$a_{21} = 6$$

$$a_{22} = 8$$

$$A \in \mathbb{R}^{2 \times 2},$$

A is 2×2 .

Vectors are kinds of matrices!

$$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^{3 \times 1}, \text{ a "column vector"}$$

$$\begin{bmatrix} 2 & 1 & 8 \end{bmatrix} \in \mathbb{R}^{1 \times 3}, \text{ a "row vector"}$$

Add'n

• Can add ^{two} matrices only when they have the same dimensions!

$$\begin{aligned} \text{e.g.,} \quad \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix} &= \begin{pmatrix} 2+1 & 4+3 \\ 6+5 & 8+7 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 7 \\ 11 & 15 \end{pmatrix} \end{aligned}$$

$$\begin{bmatrix} 2 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

but $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is nonsense.

Scalar mult. $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 7 & 8 \end{bmatrix}$

Can mult. matrices by scalars: $3 \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix} = \begin{pmatrix} 3 \cdot 2 & 3 \cdot 4 \\ 3 \cdot 6 & 3 \cdot 8 \end{pmatrix} = \begin{pmatrix} 6 & 12 \\ 18 & 24 \end{pmatrix}$

Matrices.

Try in groups:

$$A := \begin{bmatrix} 1 & 0 & 4 \\ 5 & 2 & 0 \\ 8 & 6 & 1 \end{bmatrix}, \quad B := \begin{bmatrix} 11 & 3 & 0 \\ 7 & 4 & 1 \\ 1 & 2 & 9 \end{bmatrix}$$

$$\vec{u} := \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}, \quad \vec{v} := \begin{bmatrix} 2 & 8 & 6 \end{bmatrix}, \quad \vec{w} := \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}$$

• What is a_{31} ? 8

• What is a_{13} ? 4

• What is $A + B$? $\begin{bmatrix} 12 & 3 & 4 \\ 12 & 6 & 1 \\ 9 & 8 & 10 \end{bmatrix}$

• ——— $\vec{u} + \vec{v}$? — NONSENSE!

• ——— $\vec{u} + \vec{w}$? $\begin{bmatrix} 5 \\ 9 \\ 11 \end{bmatrix}$

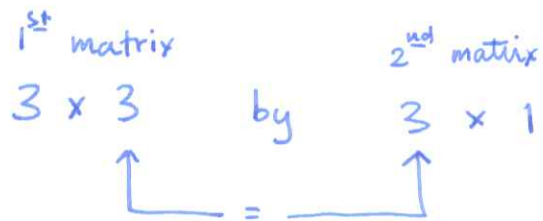
• ——— $2 \cdot A$? $\begin{bmatrix} 2 & 0 & 8 \\ 10 & 4 & 0 \\ 16 & 12 & 2 \end{bmatrix}$

• ——— $3 \cdot \vec{u}$? $\begin{bmatrix} 9 \\ 3 \\ 15 \end{bmatrix}$

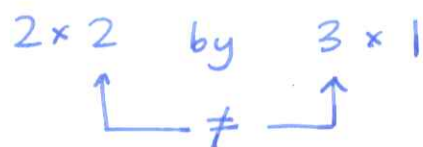
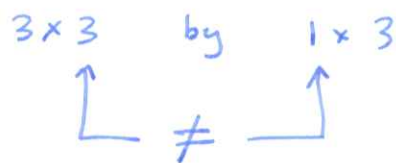
Matrix multiplication.

- Can multiply ^{two} matrices only if their "inner" dimensions match, e.g.:

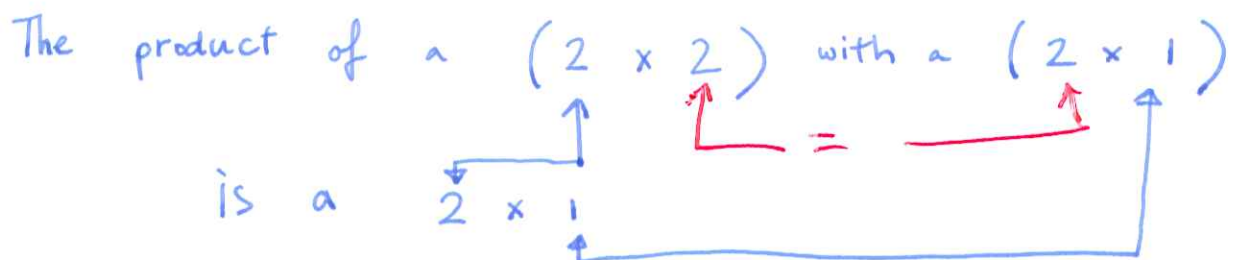
can multiply:



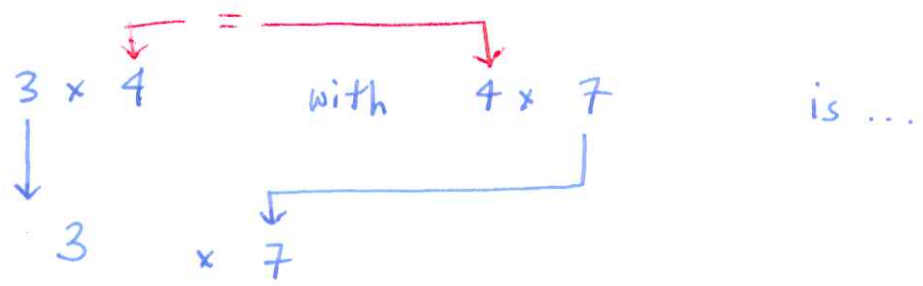
cannot multiply:



- When you multiply two matrices, the resulting product has the "outer" dimensions, e.g.:



e.g. :



1×3 with 3×1 is 1×1 (a scalar!)

Wait a minute... $(1 \times 3) \otimes_m (3 \times 1)$ gives a scalar?
 row vector \otimes_m column vector

The "matrix product" of those particular 2 vectors is a scalar.

Recall : $\hat{=}$ DOT PRODUCT. $\hat{=}$

e.g., $\langle 3, 1, 0 \rangle \cdot \langle 1, 5, 8 \rangle = \underline{3(1) + 1(5) + 0(8) = 8}$

$\langle 2, 0, 7 \rangle \cdot \langle 3, 3, 2 \rangle = \underline{20}$

In fact, $[3 \ 1 \ 0] \times \begin{bmatrix} 1 \\ 5 \\ 8 \end{bmatrix} = 8$

also!

$[2 \ 0 \ 7] \times \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} = 20$

So, we know when we're allowed to multiply two matrices, and we know the dimension of the product.

We also know that for matrices of the shapes $1 \times n$ and $n \times 1$, their product is the same as the dot product.

How to actually compute matrix products??

Let $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$.

Then $A \times B$ exists ($A \cdot B$, AB work too), because

$$(m \times n) \times (n \times p)$$

The product, call it C , is in $\mathbb{R}^{m \times p}$.

For all $i \in \mathbb{N} \cap [1, m]$ and all $j \in \mathbb{N} \cap [1, p]$,

$$C_{ij} = \underbrace{(i^{\text{th}} \text{ row of } A)}_{\text{is a vector} - (1 \times n)} \cdot \underbrace{(j^{\text{th}} \text{ col. of } B)}_{(m \times 1) \text{ vector}}$$

Example.

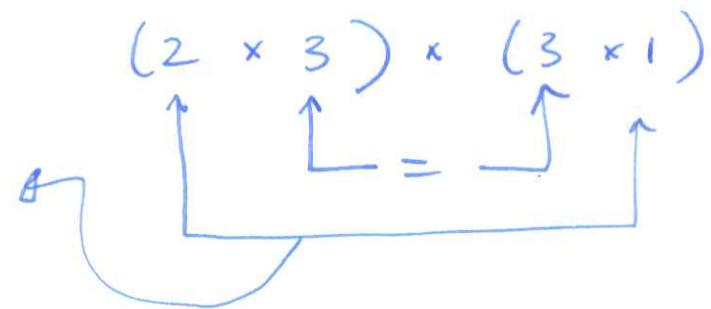
Let $A := \begin{bmatrix} 9 & 3 & 6 \\ 5 & 2 & 0 \end{bmatrix} \in \mathbb{R}^{2 \times 3}$

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$$B := \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \in \mathbb{R}^{3 \times 1}$$

Can compute the product: $(2 \times 3) \times (3 \times 1)$

$C := AB$ implies $C \in \mathbb{R}^{2 \times 1}$



$$C = \begin{bmatrix} c_{11} \\ c_{21} \end{bmatrix} = \begin{bmatrix} 21 \\ 5 \end{bmatrix}$$

$$c_{11} = (1^{\text{st}} \text{ row of } A) \cdot (1^{\text{st}} \text{ col. of } B)$$

$$= [9 \ 3 \ 6] \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 9(1) + 3(0) + 6(2) \\ = 9 + 12 = 21$$

$$c_{21} = (2^{\text{nd}} \text{ row of } A) \cdot (1^{\text{st}} \text{ col. of } B)$$

$$= [5 \ 2 \ 0] \cdot \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = 5(1) + 2(0) + 0(2) = 5$$

L2, cont'd.

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$$\text{Let } A := \begin{bmatrix} 6 & 0 & 5 & 2 \\ 9 & \del{7} & 4 & 3 \end{bmatrix} \in \mathbb{R}^{2 \times 4}$$

$$B := \begin{bmatrix} 1 & 4 \\ 2 & 0 \\ 3 & 0 \\ 6 & 0 \end{bmatrix} \in \mathbb{R}^{4 \times 2}$$

So if $C := AB$, then $C \in \mathbb{R}^{2 \times 2}$.

$$\text{Let } C := \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} = \begin{bmatrix} 33 & 24 \\ 53 & \del{32} \\ & 36 \end{bmatrix}$$

$$\begin{aligned} c_{12} &= (1^{\text{st}} \text{ row of } A) \cdot (2^{\text{nd}} \text{ col. of } B) \\ &= [6 \ 0 \ 5 \ 2] \cdot \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 24 \end{aligned}$$

Matrix transposes.

Example: If $A = \begin{bmatrix} 6 & 0 & 5 & 2 \\ 9 & 7 & 4 & 3 \end{bmatrix}$,

then $A^T = \begin{bmatrix} 6 & 9 \\ 0 & 7 \\ 5 & 4 \\ 2 & 3 \end{bmatrix}$. $(A^T)^T = A$.

In general, if $A \in \mathbb{R}^{m \times n}$, then $A^T \in \mathbb{R}^{n \times m}$,

and the i^{th} row of A^T is the same as the i^{th} col. of A , where $i \in \mathbb{N} \cap [1, m]$.