L3: Jan. 26, 2017.

Deliverables:
- WW: Tuesday
- Written: Tuesday.

today: Row reducn.

Ex. ①

$$\left.\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}\right\} \text{EQ.2} + 2\cdot\text{EQ1} \rightarrow \text{EQ2}$$

$$\left.\begin{aligned} x_1 + 5x_2 &= 7 \\ 0 + \overset{3}{\cancel{10}}\, x_2 &= 9 \end{aligned}\right\} \tfrac{1}{3}\text{EQ2} \rightarrow \text{EQ2}$$

$$\left.\begin{aligned} x_1 + 5x_2 &= 7 \\ x_2 &= 3 \end{aligned}\right\} \text{EQ1} - 5\,\text{EQ2} \rightarrow \text{EQ1}$$

$$\boxed{\begin{aligned} x_1 \quad &= -8 \\ x_2 &= 3 \end{aligned}}$$

Ex. 1
REVISIT

$$\left.\begin{aligned} x_1 + 5x_2 &= 7 \\ -2x_1 - 7x_2 &= -5 \end{aligned}\right\} \text{Equiv. To } \left(\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right).$$

$$\left(\begin{array}{cc|c} 1 & 5 & 7 \\ -2 & -7 & -5 \end{array}\right) \rightarrow \begin{matrix} R1 \\ R2+2R1 \end{matrix} \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 3 & 9 \end{array}\right) \rightarrow \begin{matrix} R1 \\ \tfrac{1}{3}R2 \end{matrix} \left(\begin{array}{cc|c} 1 & 5 & 7 \\ 0 & 1 & 3 \end{array}\right) \rightarrow$$

$$\rightarrow \begin{matrix} R1-5R2 \\ R2 \end{matrix} \left(\begin{array}{cc|c} 1 & 0 & -8 \\ 0 & 1 & 3 \end{array}\right) \text{ equiv. to } \left.\begin{aligned} 1x_1 \cdot 0 \cdot x_2 &= -8 \\ 0\cdot x_1 + 1\cdot x_2 &= 3 \end{aligned}\right\} \begin{aligned} x_1 &= -8 \\ x_2 &= 3 \end{aligned}$$

L3, ct'd.

Ex. 2 $\left.\begin{aligned}2x_1+4x_2&=-4\\5x_1+7x_2&=11\end{aligned}\right\}$ is equiv. to the augmented matrix $\left(\begin{array}{cc|c}2&4&-4\\5&7&11\end{array}\right)$

$$\left(\begin{array}{cc|c}\overset{\text{Pivot}}{\boxed{2}}&4&-4\\5&7&11\end{array}\right)\rightarrow\begin{matrix}\frac{1}{2}R1\\R2\end{matrix}\left(\begin{array}{cc|c}\overset{\text{Pivot}}{\boxed{1}}&2&-2\\5&7&11\end{array}\right)\rightarrow\begin{matrix}R1\\R2-5R1\end{matrix}\left(\begin{array}{cc|c}1&2&-2\\0&\underset{\text{Pivot}}{\boxed{-3}}&21\end{array}\right)\rightarrow$$

$$\rightarrow\begin{matrix}R1\\-\frac{1}{3}R2\end{matrix}\left(\begin{array}{cc|c}1&2&-2\\0&\underset{\text{Pivot}}{\boxed{1}}&-7\end{array}\right)\rightarrow\begin{matrix}R1-2R2\\R2\end{matrix}\left(\begin{array}{cc|c}1&0&12\\0&1&-7\end{array}\right)$$

is equiv. to the sys.

$$\left.\begin{aligned}1\cdot x_1+0\cdot x_2&=12\\0\cdot x_1+1\cdot x_2&=-7\end{aligned}\right\}\qquad\begin{aligned}x_1&=12\\x_2&=-7\end{aligned}$$

Check:

$$2(12)+4(-7)=24-28=-4\ \checkmark$$

$$5(12)+7(-7)=60-49=11\ \checkmark$$

Ex. 3

$$\left.\begin{aligned} x_1 - 4x_2 &= 1 \\ 2x_1 - x_2 &= -3 \\ -x_1 - 3x_2 &= 4 \end{aligned}\right\} \text{ equiv. to } \left(\begin{array}{cc|c} \boxed{1} & -4 & 1 \\ 2 & -1 & -3 \\ -1 & -3 & 4 \end{array}\right) \longrightarrow$$

Pivot

$$\longrightarrow \begin{array}{r} R1 \\ -2R1+R2 \\ R1+R3 \end{array} \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & \boxed{7} & -5 \\ 0 & -7 & 5 \end{array}\right) \longrightarrow \begin{array}{r} R1 \\ \frac{1}{7}R2 \\ R2+R3 \end{array} \left(\begin{array}{cc|c} 1 & -4 & 1 \\ 0 & \boxed{1} & -5/7 \\ 0 & 0 & 0 \end{array}\right) \longrightarrow$$

Pivot Pivot

$$\longrightarrow \begin{array}{r} R1+4R2 \\ R2 \\ R3 \end{array} \left(\begin{array}{cc|c} 1 & 0 & -13/7 \\ 0 & 1 & -5/7 \\ 0 & 0 & 0 \end{array}\right) \sim \begin{aligned} x_1 &= -13/7 \\ x_2 &= -5/7 \\ 0 &= 0 \end{aligned}$$

Check:

$$-\frac{13}{7} - 4\left(-\frac{5}{7}\right) = \frac{-13+20}{7} = \frac{7}{7} = 1 \ \checkmark$$

$$2\left(-\frac{13}{7}\right) - \left(-\frac{5}{7}\right) = \frac{-26+5}{7} = \frac{-21}{7} = -3 \ \checkmark$$

$$-\left(-\frac{13}{7}\right) - 3\left(-\frac{5}{7}\right) = \frac{13+15}{7} = \frac{28}{7} = 4 \ \checkmark$$

L3, ct'd.

Ex. 4 Do the planes $x_1+2x_2+x_3=4$, $x_2-x_3=1$, and $x_1+3x_2=0$ have at least one point of intersec'n?

$$\left.\begin{aligned} x_1+2x_2+x_3&=4\\ 0x_1+x_2-x_3&=1\\ x_1+3x_2+0x_3&=0\end{aligned}\right\} \sim \left(\begin{array}{ccc|c} \overset{\text{Pivot}}{\boxed{1}} & 2 & 1 & 4\\ 0 & 1 & -1 & 1\\ 1 & 3 & 0 & 0\end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 3 & 2\\ 0 & 1 & -1 & 1\\ 0 & 0 & 0 & -5\end{array}\right) \sim \left.\begin{aligned} x_1+3x_3&=2\\ x_2-x_3&=1\\ 0&=-5\end{aligned}\right\}$$

$0x_1+0x_2+0x_3=-5$ impossible!

So, no sol'n exists, and the planes don't intersect!

Ex. 5.

$$\left.\begin{aligned} x_1 + 2x_2 + x_3 &= 4 \\ x_2 - x_3 &= 1 \\ x_1 + 3x_2 \quad &= 5 \end{aligned}\right\} \sim \left(\begin{array}{ccc|c} \overset{\text{Pivot}}{\boxed{1}} & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & 5 \end{array}\right) \rightarrow$$

$$\rightarrow \begin{array}{c} R1 \\ R2 \\ R3-R1 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & \boxed{1} & -1 & 1 \\ 0 & 1 & -1 & 1 \end{array}\right) \rightarrow \begin{array}{c} R1 \\ R2 \\ R3-R2 \end{array} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 4 \\ 0 & \boxed{1} & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \rightarrow$$

$$\qquad\qquad\qquad\qquad \text{Pivot} \qquad\qquad\qquad\qquad\qquad\qquad \text{pivot}$$

$$\rightarrow \begin{array}{c} R1-2R2 \\ R2 \\ R3 \end{array} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right) \sim \left.\begin{aligned} x_1 + 3x_3 &= 2 \\ x_2 - x_3 &= 1 \\ 0 &= 0 \end{aligned}\right\}$$

So

$$\begin{aligned} x_1 &= 2 - 3x_3 \\ x_2 &= 1 + x_3 \\ x_3 &= 0 + x_3 \end{aligned} \qquad . \qquad \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

There are <u>infinitely many</u> sol'ns ! (Corresponds to infinitely many choices of x_3.)

- x_3 is called "free parameter", because in the "row-reduced" form, there is no pivot in col. #3.

L3, ct'd.

DEF. A matrix is in ROW ECHELON FORM if:

(1) any rows of all zeros appear at the bottom

(2) entries below each pivot are zero

(3) the leading entry of each pivot row is to the Ⓡ of the leading entry of the row above it.

Ex. REF:
$$\left[\begin{array}{ccccc|c} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 1 & -2 & 2 & 1 & -3 \\ 0 & 0 & 0 & 0 & -1 & 4 \end{array}\right]$$

DEF. A matrix is in REDUCED REF if, in add'n:

(1) all pivot elts. are 1

(2) each pivot elt. is the only nonzero elt. in its column.

Ex: RREF:
$$\left[\begin{array}{ccccc|c} 1 & 0 & -2 & 3 & 0 & 16 \\ 0 & 1 & -2 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & -4 \end{array}\right]$$