

L4 : Jan. 31

1) (a)

$$\begin{array}{c} \text{PIVOT} \\ \left[ \begin{array}{ccccc|c} \boxed{2} & -2 & 2 & 0 & -2 & 16 \\ 1 & 1 & 5 & 0 & 9 & 8 \\ -1 & 0 & -3 & 1 & 2 & -1 \\ 1 & 0 & 3 & 0 & 4 & 8 \end{array} \right] \end{array}$$

(b)

$$\begin{array}{c} \text{PIVOT} \\ \frac{1}{2}R_1 \\ \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array} \left[ \begin{array}{ccccc|c} \textcircled{1} & -1 & 1 & 0 & -1 & 8 \\ 1 & 1 & 5 & 0 & 9 & 8 \\ -1 & 0 & -3 & 1 & 2 & -1 \\ 1 & 0 & 3 & 0 & 4 & 8 \end{array} \right] \end{array} \rightarrow$$

$$\begin{array}{c} \text{PIVOT} \\ \begin{array}{l} R_1 \\ R_2 - R_1 \\ R_3 + R_1 \\ R_4 - R_1 \end{array} \left[ \begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -1 & 8 \\ 0 & \boxed{2} & 4 & 0 & 10 & 0 \\ 0 & -1 & -2 & 1 & 1 & 7 \\ 0 & 1 & 2 & 0 & 5 & 0 \end{array} \right] \end{array} \rightarrow$$

$$\begin{array}{c} \text{PIVOT} \\ \begin{array}{l} R_1 \\ R_4 \\ R_3 \\ R_2 \end{array} \left[ \begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -1 & 8 \\ 0 & \boxed{1} & 2 & 0 & 5 & 0 \\ 0 & \textcircled{-1} & -2 & 1 & 1 & 7 \\ 0 & \textcircled{2} & 4 & 0 & 10 & 0 \end{array} \right] \end{array} \rightarrow$$

$$\begin{array}{c} \begin{array}{l} R_1 \\ R_2 \\ R_3 + R_2 \\ R_4 - 2R_2 \end{array} \left[ \begin{array}{ccccc|c} \boxed{1} & -1 & 1 & 0 & -1 & 8 \\ 0 & \textcircled{1} & 2 & 0 & 5 & 0 \\ 0 & 0 & 0 & \boxed{1} & 6 & 7 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

REF!



$$\begin{array}{l}
 R1+R2 \\
 R2 \\
 R3 \\
 R4
 \end{array}
 \left[ \begin{array}{ccccc|c}
 1 & 0 & 3 & 0 & 4 & 8 \\
 0 & 1 & 2 & 0 & 5 & 0 \\
 0 & 0 & 0 & 1 & 6 & 7 \\
 0 & 0 & 0 & 0 & 0 & 0
 \end{array} \right]$$

RREF!

(c)

NO PIVOTS!!

So  $x_3$  and  $x_5$  are free params.

\* Any column without a pivot is a free parameter!

Let  $x_3 := s$ ,  $x_5 := t$ , for  $s, t \in \mathbb{R}$ .

$$R1: \quad x_1 + 3s + 4t = 8 \quad \Rightarrow \quad x_1 = 8 - 3s - 4t$$

$$R2: \quad x_2 + 2s + 5t = 0 \quad \Rightarrow \quad x_2 = 0 - 2s - 5t$$

$$x_3 = 0 + 1s + 0t$$

$$R3: \quad x_4 + 6t = 7 \quad \Rightarrow \quad x_4 = 7 + 0s - 6t$$

$$x_5 = 0 + 0s + 1t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 8 \\ 0 \\ 0 \\ 7 \\ 0 \end{bmatrix} + s \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} -4 \\ -5 \\ 0 \\ -6 \\ 1 \end{bmatrix}, \quad s, t \in \mathbb{R}.$$

L4, ctd.

$$\left( \begin{array}{ccc|c} \boxed{3} & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array} \right) \xrightarrow{\substack{1/3 R_1 \\ R_2 + 3R_1 \\ R_3 + 2R_1}} \left( \begin{array}{ccc|c} 1 & -4/3 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \text{ is in } \underline{\text{RREF!}}$$

Col. 2 & 3 don't have pivots, so  $x_2$  &  $x_3$  are free variables, and  $R_1$  gives the sol'n for  $x_1$  in terms of  $x_2$  &  $x_3$ :

$$x_1 - \frac{4}{3}x_2 + \frac{2}{3}x_3 = 0 \Leftrightarrow x_1 = \frac{4}{3}x_2 + \frac{2}{3}x_3$$

$$x_2 = 1x_2 + 0x_3$$

$$x_3 = 0x_2 + 1x_3$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 4/3 \\ 1 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 2/3 \\ 0 \\ 1 \end{bmatrix} x_3$$