L6: Feb. 7, 2017.

Housekeeping

- WeBWork Weds. 11:59 p.m.
- " Fri. 11:59 p.m.
- Written HW Tuesday

Last time.

Systematizing Gaussian Elimination

Deal abt. writing a computer program still stands! (see Canvas for details)

This time.

Performing the elementary row operations using matrices

~~LU-factorization (?)~~

Inverses

What is the effect of multiplying:

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}\left(\begin{array}{ccc|c}0&1&1&7\\2&0&0&4\\0&0&1&4\end{array}\right)=\left(\begin{array}{ccc|c}*&*&*&*\\ *_{21}&*_{22}&*_{23}&*_{24}\\ *&*&*&*\end{array}\right)$$

$$3\times3 \cdot 3\times4 \qquad = \left(\begin{array}{ccc|c}2&0&0&4\\0&1&1&7\\0&0&1&4\end{array}\right)$$

$*_{21}$ = (2^{nd} row)(1^{st} col.) = $[1\ 0\ 0]\begin{bmatrix}0\\2\\0\end{bmatrix} = 1\cdot0+2\cdot0+0\cdot0=0$

$*_{22}$ = (—"—)(2^{nd} ") = $[1\ 0\ 0]\begin{bmatrix}1\\0\\0\end{bmatrix} = 1\cdot1+0\cdot0+0\cdot0=1$

$*_{23}$ = (2^{nd} row)(3^{rd} col) = $[1\ 0\ 0]\begin{bmatrix}1\\0\\1\end{bmatrix} = 1\cdot1+0\cdot0+0\cdot1=1$

$*_{24}$ = (—"—)(4^{th} col) = $[1\ 0\ 0]\begin{bmatrix}7\\4\\4\end{bmatrix} = 7\cdot1+4\cdot0+4\cdot0=7$

What about multiplying:

$$\begin{pmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1\end{pmatrix}\left(\begin{array}{ccc|c}2&0&0&4\\0&1&1&7\\0&0&1&4\end{array}\right)=\left(\begin{array}{ccc|c}1&0&0&2\\0&1&1&7\\0&0&1&4\end{array}\right)$$

In general, to scale a row of a matrix, multiply on the left by the identity matrix, slightly modified (scale the corresponding row of the identity by the same scalar).

E.G.,
$$\begin{pmatrix}\frac{1}{2}&0&0\\0&1&0\\0&0&1/4\end{pmatrix}\begin{pmatrix}2&1&1\\1&1&1\\4&1&2\end{pmatrix}=\begin{pmatrix}1&\frac{1}{2}&\frac{1}{2}\\1&1&1\\1&\frac{1}{4}&\frac{1}{2}\end{pmatrix}$$

Or multiplying:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 4 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$*_{21} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = 0\cdot 1 + 1\cdot 0 - 1\cdot 0 = 0$$

$$*_{22} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = 0\cdot 0 + 1\cdot 1 - 1\cdot 0$$

$$*_{23} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0\cdot 0 + 1\cdot 1 - 1\cdot 1$$

$$*_{24} = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 7 \\ 4 \end{bmatrix} = 0\cdot 2 + 1\cdot 7 + (-1)\cdot 4 = 3$$

What about the entire product?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|c} 0 & 1 & 1 & 7 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right)} =$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|c} 2 & 0 & 0 & 4 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 4 \end{array}\right)}$$

$$= \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 1 & 4 \end{array}\right)}$$

$$= \left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{array}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 7 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right) =$$

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1/2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 7 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right) =$$

$$\begin{pmatrix} 0 & 1/2 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & 1 \end{pmatrix} \left(\begin{array}{ccc|c} 0 & 1 & 1 & 7 \\ 2 & 0 & 0 & 4 \\ 0 & 0 & 1 & 4 \end{array}\right) =$$

$$=$$