MATH 330 · Calculus III
Autumn 2016
Instructor: E.M. Kiley

Homework 1: Due in class September 14

How This Works

Please find each class's reading assignments on the syllabus. These should be completed *before* lecture on the dates indicated. In mathematics, reading without solving problems is useless—so there will be a small number of computationally-based problems on Canvas for you to complete on most class days. However, another important part of transitioning to higher-level mathematics is developing the ability to think critically about problems you haven't seen before. Each week, you will therefore be expected to complete a small number of supplementary problems that are less trivial in nature than the ones you'll submit on Canvas. You are expected to complete these problems with great care and to present them in a professional way. You are encouraged to type your solutions using LaTeX, or to handwrite them very neatly; if your work is not presentable or if it is illegible, you will not receive credit for it.

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. **[15 points]** Use the "epsilon" definition of convergence to prove that $\lim_{n\to\infty} \dfrac{\sin n}{n} = 0$. Hint: $|\sin(x)| < 1$ for all x.

Problem 2. Newton's method, applied to a differentiable function $f(x)$, begins with a starting value x_0 and generates from it a sequence of numbers $\{x_n\}$ that, under favorable circumstances, converges to a zero of f. The recursion formula for the sequence is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

(a) **[7 points]** Show that the recursion formula for $f(x) = x^2 - a$, for $a > 0$ constant, can be written as $x_{n+1} = \dfrac{x_n + \frac{a}{x_n}}{2}$.

(b) **[8 points]** Starting with $x_0 = 1$ and $a = 3$, calculate successive terms of the sequence until the display on your calculator or computer prompt begins to repeat. What number is being approximated? Please refer to the function $f(x)$ in your answer.

Problem 3. **[15 points]** Let $\{F_n\}$ be the Fibonacci sequence as defined in the first class, and assume that

$$\tau := \lim_{n\to\infty} \frac{F_{n+1}}{F_n}$$

exists (it does). Show that

$$\tau = \frac{1+\sqrt{5}}{2}.$$

[Hint: Define a sequence $a_n := \frac{F_n}{F_{n-1}}$ and show that $a_{n+1} = 1 + \frac{1}{a_n}$. Make sure you use the assumption that τ exists, and explicitly state where in your work you needed that assumption. You will also need to use the fact that $\tau \neq 0$. Why is this true, and where do you use it?]

Problem 4. **[15 points]** Determine whether the sequence with terms $\left(1 + \frac{2}{n}\right)^{2n}$ converges, and find its limit if it does converge. [Hint: Use l'Hôpital's Rule to evaluate the limit.]

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`