MATH 330 · Calculus III
Autumn 2016
Instructor: E.M. Kiley

Homework 2: Due in class September 22

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

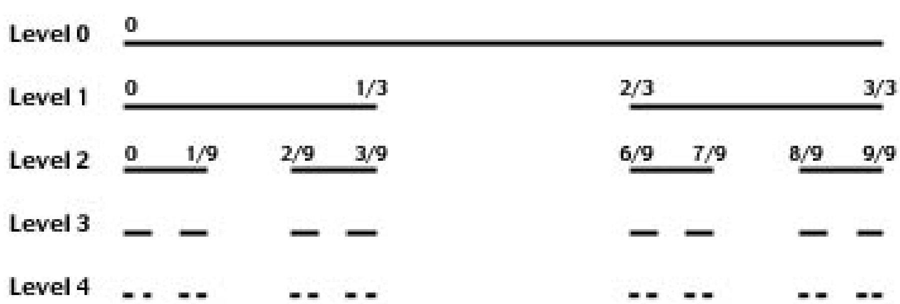
Problem 1. **[10 points]** For which values of x does the geometric series

$$\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n\left(\frac{1}{2+\sin x}\right)^n$$

converge? What is the sum when the series does converge? [Hint: what is r? Express the inequality $|r|<1$ in terms of x. The sum should also necessarily involve x.]

Problem 2. This problem explores the Cantor set. To construct this set, we begin with the closed interval $[0,1]$. From that interval, remove the middle open interval $(1/3,-1/3)$, leaving two closed intervals $[0,1/3]$ and $[2/3,1]$. At the second step, remove the open middle third interval from each of those remaining—*i.e.*, from $[0,1/3]$ we remove the open interval $(1/9,2/9)$ and from $[2/3,1]$ we remove $(7/9,8/9)$, leaving behind the four closed intervals $[0,1/9]$, $[2/9,1]$, $[2/3,7/9]$, and $[8/9,1]$. At the next step, we remove the middle open third interval from each closed interval left behind, so $(1/27,2/27)$ is removed from $[0,1/9]$, leaving the closed intervals $[0,1/27]$ and $[2/27,1/9]$; $(7/27,8/27)$ is removed from $[2/9,7/27]$, leaving behind $[2/9,7/27]$ and $[8/27,1/3]$; and so forth.

We continue this process repeatedly without stopping, at each step removing the open third interval from the middle of ever closed interval remaining after the previous step. The numbers remaining in the interval $[0,1]$, after all open middle third intervals have been removed, are the points in the Cantor set (named after Georg Cantor, 1845–1918)[2]. A picture of this process is below. The set has some interesting properties.



(a) **[3 points]** The Cantor set contains infinitely many numbers in $[0,1]$. One of them is $1/3$ (this number is never removed in the iterations). List four other numbers that belong to the Cantor set.

(b) **[7 points]** At the first step, the length of the interval removed from $[0,1]$ is $1/3$. At the second step, the total length of the intervals removed is $2\cdot(1/9)=2/9$. At the third step, the total length of the intervals removed is $4\cdot(1/27)=4/27$, and so forth. Write the correct geometric series that describes the total length of all the open middle third intervals that have been removed, and if it converges, find its sum.

[1] See a list of mathematical symbols and their meanings here: `http://en.wikipedia.org/wiki/List_of_mathematical_symbols`

[2] For more information about the Cantor set, including more pictures representing the various stages of its construction, please see `http://personal.bgsu.edu/~carother/cantor/Cantor1.html` , and remember to click the 'more on the Cantor set' link at the bottom of the page to continue reading.

Problem 3. **[10 points]** As you know, **the harmonic series does not converge** (the harmonic series does not converge). But its partial sums just grow so, so slowly that there is no empirical evidence for this fact; suppose, in fact, that we had started computing the partial sums with $s_1 = 1$ the day the universe was formed, 13 billion years ago, and that we added a new term *every second*. About how large would the partial sum s_n be today, assuming that all years had 365 days, and that leap seconds (like the one we had in 2015!) did not exist?

[Hint: Compute the number of seconds since the day the universe was formed, under the assumptions listed. This is the upper limit of the partial sum you seek. Then use an online tool like Wolfram Alpha, or write a short computer program in the language of your choice (e.g., Python, MATLAB, etc.), to compute the finite sum

$$\sum_{n=1}^{\text{number of seconds}} \frac{1}{n}.$$

Please show your work, including how you arrived at the number of seconds, and show which tool(s) you used to compute the partial sum. Include screenshots, if you can. If you wrote a computer program, I want to see it!]