MATH 330 · Calculus III
Autumn 2016
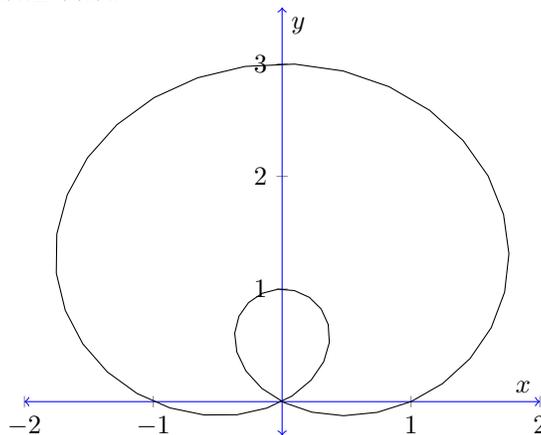
<div align="center">**Homework 5: Due in class November 2**</div>

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words[1], and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the "running equals sign", as this is an abuse of notation and is unacceptable: `http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage`. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. The curve with parametric equations

$$x = (1 + 2\sin\theta)\cos\theta, \qquad y = (1 + 2\sin\theta)\sin\theta$$

is called a *limaçon* and is shown below[2].

Find the points (x, y) and the slopes of the tangent lines at these points for the following values of the parameter θ:

   (a) [**3 points**] $\theta = 0$.

   (b) [**3 points**] $\theta = \pi/2$.

   (c) [**4 points**] $\theta = 4\pi/3$. For this θ value, please also write an equation for the line tangent to the limaçon at the point (x, y). (The equation of the tangent line can be in Cartesian coordinates—no need to parametrize.)

Problem 2. To illustrate the fact that the numbers we get for curve length do not depend on the way we parametrize our curves (except for the restrictions preventing doubling back mentioned in the definition of the arc length), calculate the length of the semicircle $y = \sqrt{1 - x^2}$ with these two different parametrizations:

   (a) [**5 points**] $x = \cos(2t), \quad y = \sin(2t), \quad t \in [0, \pi/2]$.

   (b) [**5 points**] $x = \sin(\pi t), \quad y = \cos(\pi t), \quad t \in [-1/2, 1/2]$.

Problem 3. [**10 points**] Find the area under one arch of the cycloid

$$x = a(t - \sin t), \qquad y = a(1 - \cos t).$$