9 Dec. 2016, 1

Housekeeping: Homework due Monday

Monday w/b review for final exam

You can use an 8.5" × 11" sheet of notes (both sides) on final exam

Final exam is W, Dec. 14 at 1 p.m. here

Last time:
- Dot prod.
- Cross prod.

Q.: $(a\vec{u}) \times \vec{v} = ?$

$\vec{u} := \langle u_1, u_2, u_3 \rangle$

$a\vec{u} = \langle au_1, au_2, au_3 \rangle$

$\vec{v} := \langle v_1, v_2, v_3 \rangle$

$$(a\vec{u}) \times \vec{v} = \det \begin{pmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ au_1 & au_2 & au_3 \\ v_1 & v_2 & v_3 \end{pmatrix}$$

$$= \hat{\imath} \det \begin{pmatrix} au_2 & au_3 \\ v_2 & v_3 \end{pmatrix} - \hat{\jmath} \det \begin{pmatrix} au_1 & au_3 \\ v_1 & v_3 \end{pmatrix} + \hat{k} \det \begin{pmatrix} au_1 & au_2 \\ v_1 & v_2 \end{pmatrix}$$

$$= \hat{\imath}(au_2 v_3 - au_3 v_2) - \hat{\jmath}(au_1 v_3 - au_3 v_1) + \hat{k}(au_1 v_2 - au_2 v_1)$$

$$= a\left[\hat{\imath}(u_2 v_3 - u_3 v_2) - \hat{\jmath}(u_1 v_3 - u_3 v_1) + \hat{k}(u_1 v_2 - u_2 v_1)\right]$$

$$= a(\vec{u} \times \vec{v})$$

$$\boxed{(a\vec{u}) \times \vec{v} = a(\vec{u} \times \vec{v}) = \vec{u} \times (a\vec{v})}$$

Exercise. Find a vector $\perp$ to the plane that contains $P(1,-1,0)$, $Q(2,1,-1)$, and $R(-1,1,2)$.

Strategy: Find two vectors in the plane, & take their cross product.

$$\overrightarrow{PQ} = \langle 2-1,\ 1-(-1),\ -1-0 \rangle = \langle 1, 2, -1 \rangle$$

$$\overrightarrow{PR} = \langle -1-1,\ 1-(-1),\ 2-0 \rangle = \langle -2, 2, 2 \rangle = 2\langle -1, 1, 1 \rangle$$

A vector normal to both $\overrightarrow{PQ}$ & $\overrightarrow{PR}$ will be $\perp$ to the plane containing P, Q, & R.

$\overrightarrow{PQ} \times \overrightarrow{PR}$ is just such a vector (by def'n of cross prod.)

$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} :=: \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{vmatrix} = \hat{\imath}\begin{vmatrix} 2 & -1 \\ 2 & 2 \end{vmatrix} - \hat{\jmath}\begin{vmatrix} 1 & -1 \\ -2 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 2 \\ -2 & 2 \end{vmatrix}$$

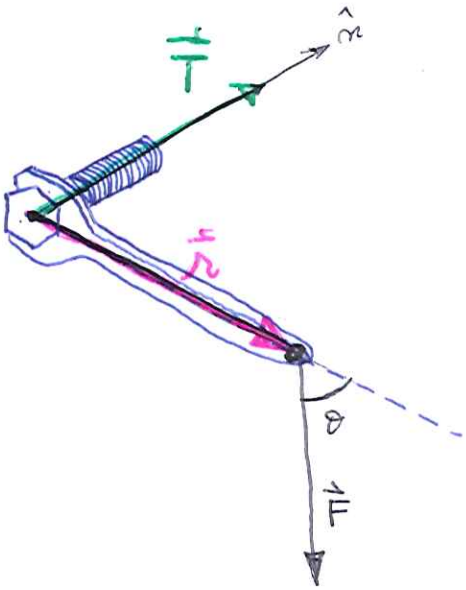
$$= \hat{\imath}\left(2\cdot 2 - 2(-1)\right) - \hat{\jmath}\left(1(2) - (-1)(-2)\right) + \hat{k}\left(1(2) - (2)(-2)\right)$$

$$= \hat{\imath}\,(4+2) - \hat{\jmath}\,(2-2) + \hat{k}\,(2+4)$$

$$= 6\hat{\imath} + 0\hat{\jmath} + 6\hat{k} = \langle 6, 0, 6 \rangle.$$

The unit vector in this direc'n is $\dfrac{\langle 6,0,6\rangle}{\sqrt{6^2+0^2+6^2}} = \left\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle.$

Torque.



<u>Torque vector</u> that results when we apply force $\vec{F}$ to turn a bolt, $\vec{r}$ away from the bolt's head, is:

$$\vec{\tau} = \vec{r} \times \vec{F}$$
$$= \left(|\vec{r}||\vec{F}|\sin\theta \right) \hat{n}$$

<u>Note</u>: $$|\vec{\tau}| = \left| \underbrace{|\vec{r}||\vec{F}|\sin\theta}_{\text{scalar}} \, \hat{n} \right| = \overbrace{\Big| |\vec{r}||\vec{F}|\sin\theta \Big|}^{\text{abs. val.}} \underbrace{\overbrace{|\hat{n}|}^{\text{mag.}}}_{=1}$$
$$= |\vec{r}||\vec{F}|\,|\sin\theta|$$

The 3 ways of increasing torque's magnitude:

- Increasing $|\vec{F}|$ (push harder)
- Increasing $|\sin\theta|$ (push in the right direc'n — $\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$)
- Increase $|\vec{r}|$