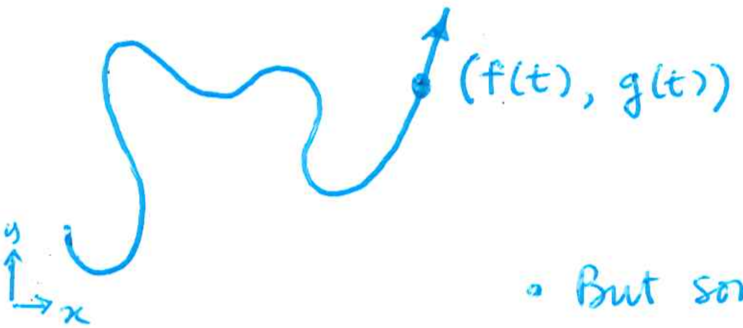
Oct. 19 : Parametric Curves.



- This path is not the graph of a function of the variable x.

- But sometimes, the path can be described by writing its coordinates as $x = f(t)$, $y = g(t)$ for cts. fns. f and g. So, at time t, the particle is at $(x, y) = (f(t), g(t))$.

DEF'N. If x and y are given as fns. $x = f(t)$, $y = g(t)$ over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these eq'ns is a PARAMETRIC CURVE. The eq'ns are PARAMETRIC EQ'NS for the curve.

- The variable t is called a PARAMETER for the curve.
- I is called the PARAMETER INTERVAL. If $I = [a, b]$ (i.e., if $a \leq t \leq b$), then we call $(f(a), g(a))$ the INITIAL POINT of the curve, and we call $(f(b), g(b))$ the TERMINAL POINT.

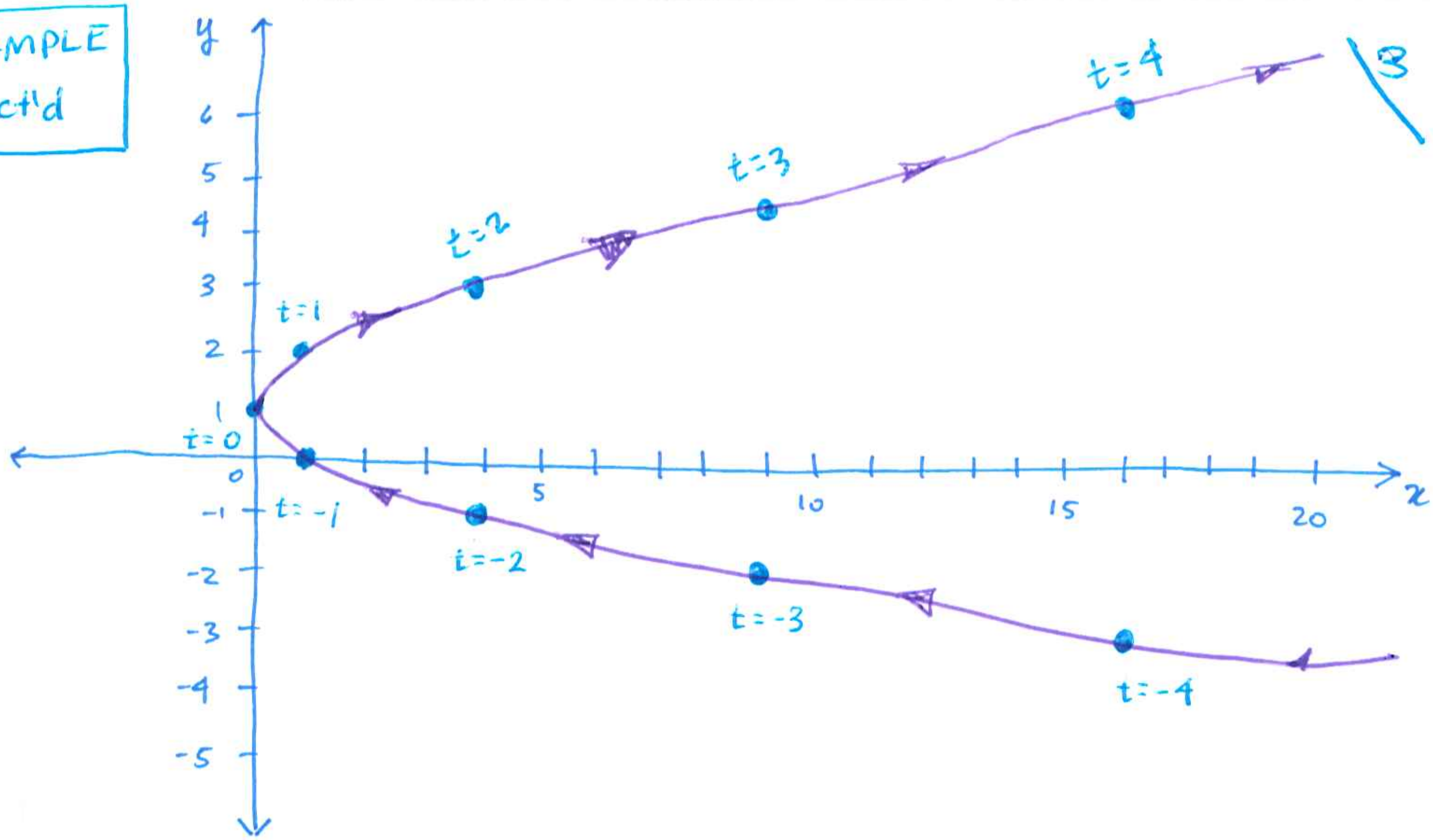
- The parametric eq'ns, together with the interval I, constitute a <u>PARAMETRIZATION</u> of the curve, and when we give a parametrization, we say we've <u>PARAMETRIZED</u> the curve.

<u>EXAMPLE 1</u>. Sketch the curve defined by the parametric eq'ns: $x = t^2$, $y = t + 1$, $-\infty < t < \infty$.

Each value of t gives a point (x, y) on the curve. So just compute several points on the curve, plot them, and connect with a smooth curve.

t	$x = t^2$	$y = t + 1$	(x, y)
-5	25	-4	$(25, -4)$
-4	16	-3	$(16, -3)$
-3	9	-2	$(9, -2)$
-2	4	-1	$(4, -1)$
-1	1	0	$(1, 0)$
0	0	1	$(0, 1)$
1	1	2	$(1, 2)$
2	4	3	$(4, 3)$
3	9	4	$(9, 4)$
4	16	5	$(16, 5)$
5	25	6	$(25, 6)$

EXAMPLE 1, ct'd



- Computed points ; • Plotted pts ; • Connected pts ;
- Drew arrows in the direc'n from lower t-vals. to higher t-values.

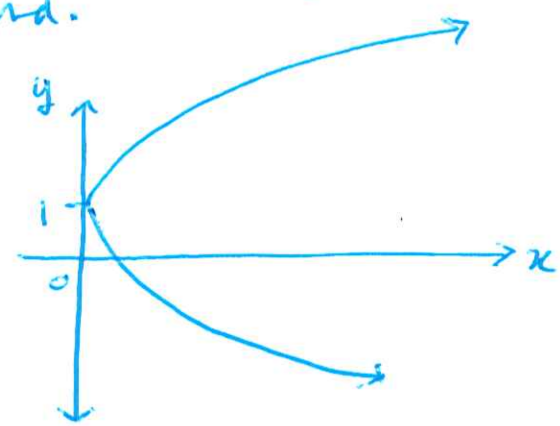
* "Particle" travels faster when the distance btwn. fixed time intervals is larger.

Eliminating the parameter.

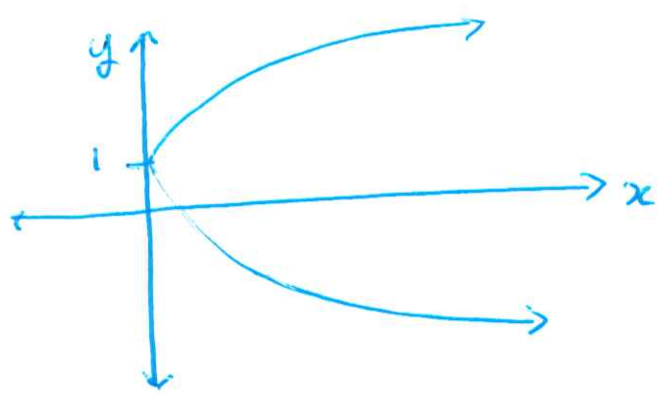
EXAMPLE 2. Eliminate the parameter t and identify the curve: $x = t^2$, $y = t+1$, $-\infty < t < \infty$.

Well, $x = t^2$ and $y = t+1$. Solve for t in terms of y: $t = y-1$, so $x = (y-1)^2 = y^2 - 2y + 1$.

This is the eq'n of a parabola in y. Its vertex will be at the point $(0,1) = (x,y)$, and it will open rightward.



Alternatively: If $x = t^2$, then $t = \pm\sqrt{x}$, so $y = t+1$ implies $y = \pm\sqrt{x} + 1 = 1 \pm \sqrt{x}$.

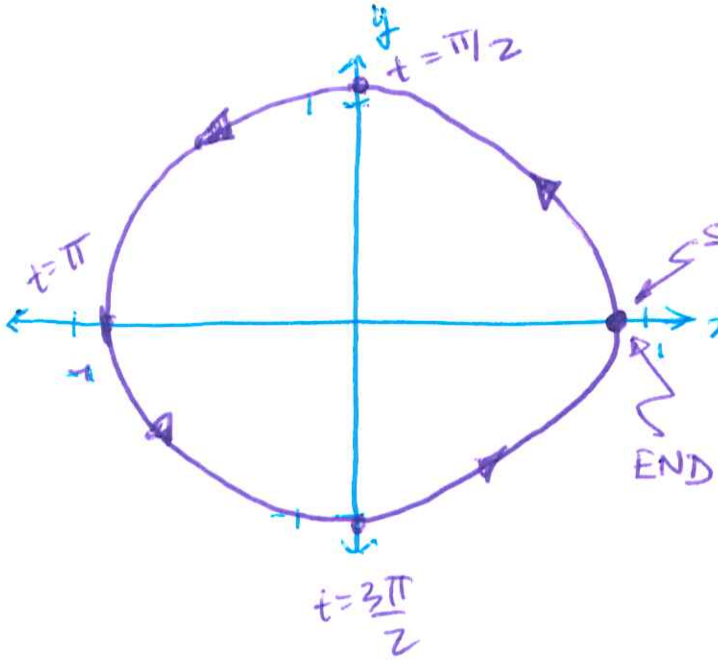


EXAMPLE 3 Graph the parametric curve:

$$\begin{cases} x = \cos(t) \\ y = \sin(t) \\ 0 \le t \le 2\pi \text{ , i.e., } t \in [0, 2\pi] \end{cases}$$

Familiar? $x^2 + y^2 = \cos^2 t + \sin^2 t = 1$.

This is the graph of a circle, w. radius 1, centered at $(0,0)$.



Starting pt. at $t=0$:
$(\cos(0), \sin(0)) = (1,0)$

Terminal pt. at $t = 2\pi$:
$(\cos(2\pi), \sin(2\pi)) = (1,0)$

EXAMPLE 4

$$\begin{cases} x = a\cos(t) + c \\ y = a\sin(t) + d \\ 0 \le t \le 2\pi \end{cases}$$

$(x-c)^2 + (y-d)^2 = a^2$

$$\underbrace{(x-0)^2 + (y-0)^2}_{\text{ctr. of circle}} = \underset{\text{rad. of circle.}}{a^2}$$

Observe: $x^2 + y^2 = a^2\cos^2 t + a^2\sin^2 t$
$= a^2(\cos^2 t + \sin^2 t)$
$= a^2$

$x^2 + y^2 = a^2$ is a circle of rad. a, centered at $(0,0)$.