2 Dec.

Housekeeping: • HW 8 due Weds. in class

• Final Exam : Weds, Dec. 14 @ 1 p.m.
in the usual classroom.

Last time: • unit vectors
• magnitude & direc'n
• properties of vec. add'n & scalar multiplicat'n.

Warm-up: If $\vec{v} := 3\hat{\imath} - 4\hat{\jmath}$ is a velocity vector, express $\vec{v}$ as the product of its speed and its direc'n of motion.

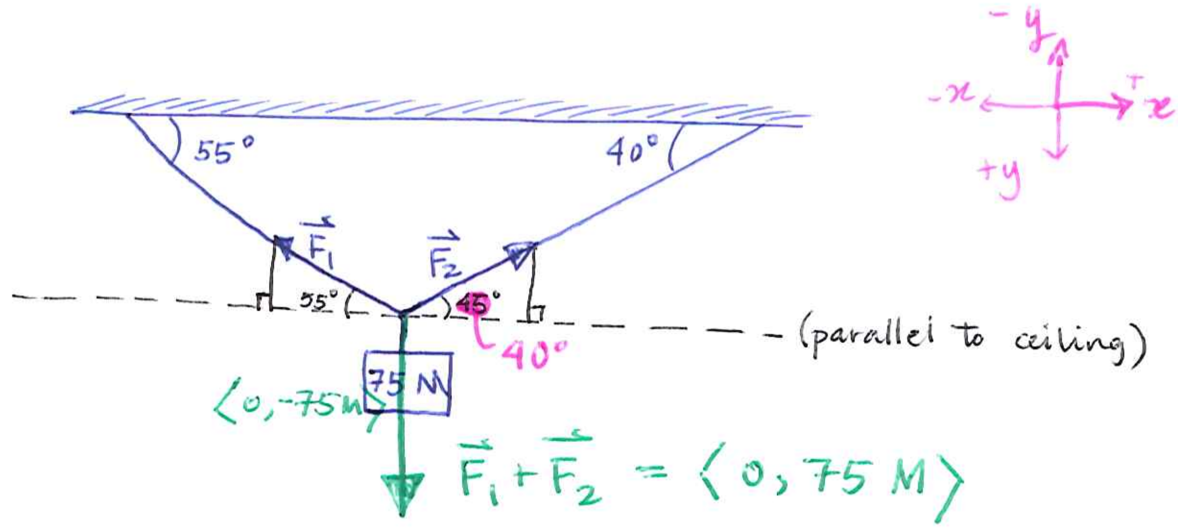
Hints: • speed is the magnitude of velocity
• direc'n is always a unit vector.

Speed
$= \text{Magnitude} = \sqrt{(3)^2 + (-4)^2} = \sqrt{25} = 5$ Jordans/seconds

$\frac{1}{5}(3\hat{\imath} - 4\hat{\jmath}) = \frac{3}{5}\hat{\imath} - \frac{4}{5}\hat{\jmath} + 0\hat{k} = \left\langle \frac{3}{5}, -\frac{4}{5}, 0 \right\rangle$

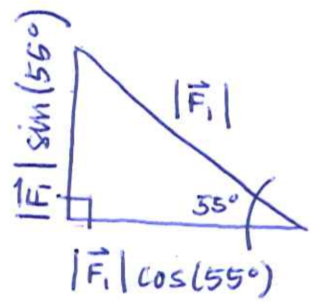
So $\vec{v} = \underbrace{5}_{\text{speed}} \cdot \underbrace{\left\langle \frac{3}{5}, -\frac{4}{5}, 0 \right\rangle}_{\text{direc'n.}}$

Example. A 75-N weight is suspended by two wires as shown. Find the forces $\vec{F}_1$ and $\vec{F}_2$ acting in both wires:

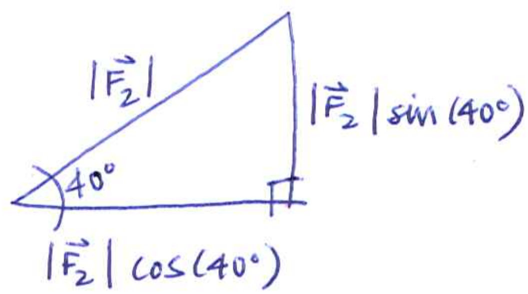


$$\vec{F}_1 + \vec{F}_2 = \langle 0, 75 N \rangle$$

Note:



and



So $\vec{F}_1 = -|\vec{F}_1| \cos(55°)\,\hat{i} + |\vec{F}_1| \sin(55°)\,\hat{j} = \langle -|\vec{F}_1| \cos 55°, |\vec{F}_1| \sin 55° \rangle$

$+ \quad \vec{F}_2 = |\vec{F}_2| \cos(40°)\,\hat{i} + |\vec{F}_2| \sin(40°)\,\hat{j}$

$$0\,\hat{i} \quad + \quad 75 N\,\hat{j}$$

$$-|\vec{F}_1| \cos(55°) + |\vec{F}_2| \cos(40°) = 0 \qquad |\vec{F}_1| \sin(55°) + |\vec{F}_2| \sin(40°) = 75 N$$

$$|\vec{F}_1| = \frac{|\vec{F}_2| \cos(40°)}{\cos(55°)}$$

$$|\vec{F_2}| \frac{\cos(40^\circ)}{\cos(55^\circ)} \sin(55^\circ) + |\vec{F_2}| \sin(40^\circ) = 75\ M$$

$$|\vec{F_2}| \left(\frac{\cos(40^\circ)\sin(55^\circ)}{\cos(55^\circ)} + \sin(40^\circ) \right) = 75\ M$$

$$|\vec{F_2}| \left(\cos(40^\circ)\tan(55^\circ) + \sin(40^\circ) \right) = 75\ M$$

$$|\vec{F_2}| = \frac{75\ M}{\cos(40^\circ)\tan(55^\circ) + \sin(40^\circ)} \approx 43.2\ M$$

$$|\vec{F_1}| = |\vec{F_2}| \frac{\cos(40^\circ)}{\cos(55^\circ)} \approx 57.7\ M$$

Finally,

$$\boxed{\vec{F_1} = \langle -|\vec{F_1}|\cos(55^\circ),\ |\vec{F_1}|\sin(55^\circ) \rangle \approx \langle -33.08,\ 47.24 \rangle}$$

$$\boxed{\vec{F_2} = \langle |\vec{F_2}|\cos(40^\circ),\ |\vec{F_2}|\sin(40^\circ) \rangle \approx \langle 33.08,\ 27.76 \rangle}$$

$$\vec{F_1} + \vec{F_2} \approx \langle -33.08 + 33.08,\ 47.24 + 27.76 \rangle$$

$$= \langle 0,\ 75\,M \rangle \ \checkmark$$

The dot product.

DEF. If $\vec{u} := \langle u_1, u_2 \rangle$, $\vec{v} := \langle v_1, v_2 \rangle$, then

$$\vec{u} \cdot \vec{v} := u_1 v_1 + u_2 v_2 \, , \quad \text{and} \ldots$$

$$\vec{u} \cdot \vec{v} := \text{———} '' \text{———} + u_3 v_3 \quad \text{if } \vec{u} := \langle u_1, u_2, u_3 \rangle, \ \vec{v} := \langle v_1, v_2, v_3 \rangle$$

→ THIS IS A SCALAR. ←

THE DOT PRODUCT OF TWO VECTORS IS A <u>SCALAR</u>.

(THIS IS WHY THE DOT PRODUCT IS SOMETIMES CALLED THE "<u>SCALAR</u> PRODUCT")

Example. $\langle 1, -2, 1 \rangle \cdot \langle -6, 2, -3 \rangle =$

$$= 1(-6) + (-2)(2) + 1(-3)$$

$$= -6 - 4 - 3 = -13.$$

$$\left(\tfrac{1}{2}\hat{\imath} + 3\hat{\jmath} + \hat{k}\right) \cdot \left(4\hat{\imath} - \hat{\jmath} + 2\hat{k}\right) = \tfrac{1}{2}(4) + 3(-1) + 1(2)$$

$$= 2 - 3 + 2 = 1.$$