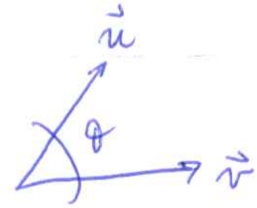
5 DEC '16

House keeping:
- HW 8 due W in class
- Final: W Dec 14 @ 1 p.m., this classroom.

Last time:
- Velocity
- Force balancing
- Dot product

In $\mathbb{R}^2$; $\mathbb{R}^3$, the angle btwn. two vectors $\vec{u} := \langle u_1, u_2 \rangle$ and $\vec{v} := \langle v_1, v_2 \rangle$, is

$$\theta = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|\,|\vec{v}|}\right)$$



Warm-up: Find the angle btwn. $\vec{u} := \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{v} := 6\hat{i} + 3\hat{j} + 2\hat{k}$.

$\vec{u} \cdot \vec{v} = 1(6) + (-2)(3) + (-2)(2)$

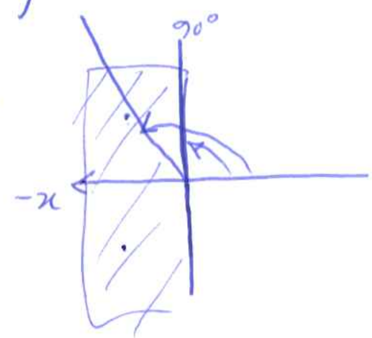
$= 6 - 6 - 4 = -4$

$|\vec{u}| = \sqrt{1^2 + (-2)^2 + (-2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$|\vec{v}| = \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36+9+4} = \sqrt{49} = 7$

so $\theta = \arccos\left(\dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right) = \arccos\left(\dfrac{-4}{3 \cdot 7}\right) = \arccos\left(\dfrac{-4}{21}\right).$

≈ 1.76 radians or $100.98°$

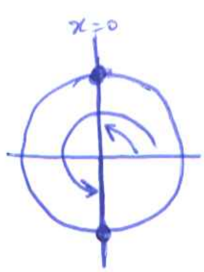


If the ∡ btwn. two vectors is

$$\theta = \arccos\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right),$$

we say two vectors are <u>orthogonal</u> when $\vec{u} \cdot \vec{v} = 0$.

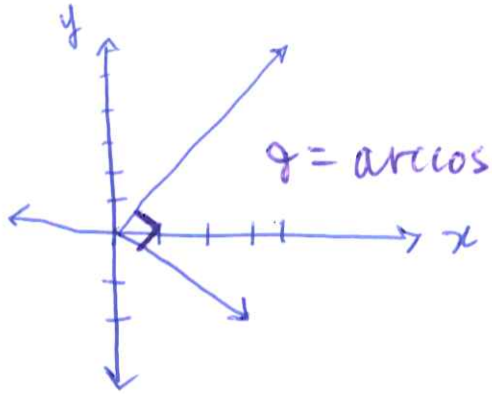
<u>Why?</u> $\vec{u} \cdot \vec{v} = 0$ implies $\theta = \arccos\left(\frac{0}{|\vec{u}||\vec{v}|}\right) = \arccos(0)$



and $\arccos(0) = \frac{\pi}{2}$ or $\frac{3}{2}$, etc.

$= 90°, 270°,$ etc.

Ex: Are $\vec{u} := \langle 3,-2 \rangle$ and $\vec{v} := \langle 4,6 \rangle$ orthogonal?



$$\theta = \arccos\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right)$$

$\vec{u}\cdot\vec{v} = 3(4) + (-2)(6)$

$= 12 - 12 = 0.$

so $\arccos(0) = \theta$

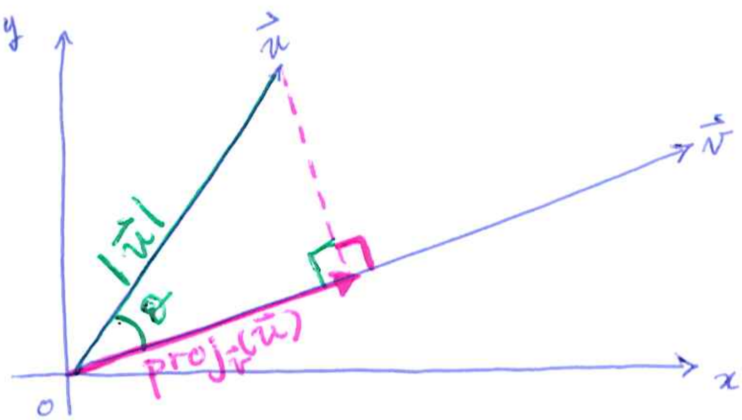
so $\theta = 90° = \frac{\pi}{2}$.

Ex: Are $\vec{0}$ and $\vec{u}$ orthogonal?

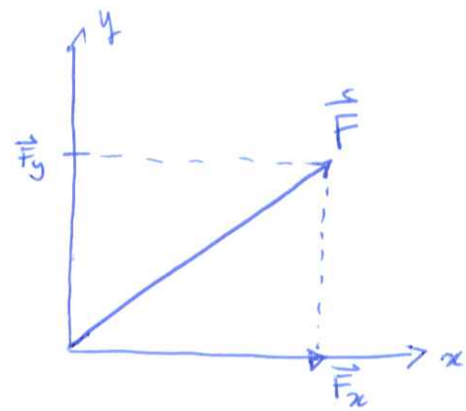
$$\vec{u}\cdot\vec{0} = 0(u_1) + 0(u_2) + 0(u_3) = 0.$$

Yes, the zero vector is orthogonal to everything, incl. itself.

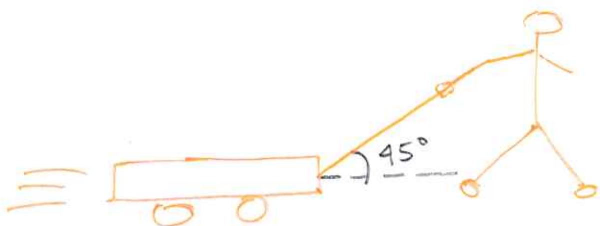
Vector projec'n.



$\text{proj}_{\vec{v}}(\vec{u})$



recall:



$$\vec{F}_x = \text{proj}_{\hat{\imath}}(\vec{F})$$

$$\vec{F}_y = \text{proj}_{\hat{\jmath}}(\vec{F})$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$$

$$\cos\theta = \frac{|\text{proj}_{\vec{v}}(\vec{u})|}{|\vec{u}|}$$

so $$\frac{\vec{u}\cdot\vec{v}}{\cancel{|\vec{u}|}|\vec{v}|} = \frac{|\text{proj}_{\vec{v}}(\vec{u})|}{\cancel{|\vec{u}|}}$$

$$|\text{proj}_{\vec{v}}(\vec{u})| = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}$$

so $$\boxed{\text{proj}_{\vec{v}}(\vec{u}) = \underbrace{\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|}\right)}_{\text{magnitude}}\underbrace{\left(\frac{\vec{v}}{|\vec{v}|}\right)}_{\text{direc'n}} = \underbrace{\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)}_{\text{scalar}}\underbrace{\vec{v}}_{\text{vector}}}$$

Ex. Find the vector projec'n of $\vec{u} := 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\vec{v} := \hat{i} - 2\hat{j} - 2\hat{k}$.

$$\mathrm{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v}$$

$$\vec{u}\cdot\vec{v} = 6(1) + 3(-2) + 2(-2) = -4$$

$$|\vec{v}|^2 = 1^2 + (-2)^2 + (-2)^2 = 9.$$

So $\dfrac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2} = -\dfrac{4}{9}$. So $\mathrm{proj}_{\vec{v}}(\vec{u}) = \left(-\dfrac{4}{9}\right)\langle 1, -2, -2\rangle$

$$= \left\langle -\frac{4}{9}, \frac{8}{9}, \frac{8}{9}\right\rangle.$$

$$\mathrm{proj}_{\vec{u}}(\vec{v}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}|^2}\right)\vec{u} = \left(\frac{-4}{49}\right)\langle 6, 3, 2\rangle = \left\langle \frac{-24}{49}, \frac{-12}{49}, \frac{-8}{49}\right\rangle$$

$$|\vec{u}|^2 = (6)^2 + 3^2 + 2^2 = 36 + 9 + 4 = 49.$$