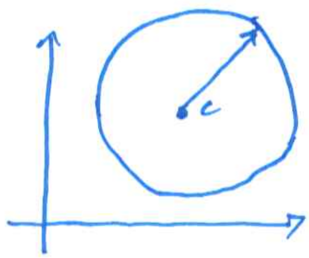
Nov. 21 : 3D coordinate systems & vectors.

<u>Reminder</u>: Written HW due Monday after break.

<u>Last time</u>: 3D coord. systems.

<u>Reminder</u>: In 2D, a circle of radius r and center c is the set of all points of distance r away from the point c.



$$\text{circle}\left(\overbrace{(a,b)}^{c}, r\right) = \{(x,y) : |(a,b),(x,y)| = r\}$$

$$= \{(x,y) : \underbrace{(x-a)^2 + (y-b)^2}_{\text{dist. in 2D}} = r^2\}$$

<u>In 3D</u>, a sphere (spherical shell) of radius r & ctr. (a,b,c) is the set of all pts. of dist. r from (a,b,c).

$$\text{Sphere}\left((a,b,c), r\right) = \{(x,y,z) : |(a,b,c) \,\&\, (x,y,z)| = r\}$$

$$= \boxed{\{(x,y,z) : (x-a)^2 + (y-b)^2 + (z-c)^2 = r^2\}}$$

The eq'n of a sphere is $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$.

<u>EXAMPLE ①</u>. Find the ctr. & radius of

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0.$$

<u>Strategy</u>: complete the square in x, y, & z.

$$\left[x^2+3x\right]+\left[y^2\right]+\left[z^2-4z\right]=-1$$

add $\left(\frac{3}{2}\right)^2$ to both sides

$(x+a)^2 = x^2+2ax+a^2$

add $\left(\frac{-4}{2}\right)^2$ to both sides

$$\left[x^2+3x+\left(\tfrac{3}{2}\right)^2\right]+\left[y^2\right]+\left[z^2-4z+\left(\tfrac{-4}{2}\right)^2\right]=-1+\left(\tfrac{3}{2}\right)^2+\left(\tfrac{-4}{2}\right)^2$$

$$\left(x+\tfrac{3}{2}\right)^2+(y+0)^2+\left(z-\tfrac{4}{2}\right)^2=-1+\tfrac{9}{4}+4$$

$$\left(x+\tfrac{3}{2}\right)^2+(y+0)^2+(z-2)^2=3+\tfrac{9}{4}=\tfrac{12+9}{4}=\tfrac{21}{4}.$$

$$\left(x+\tfrac{3}{2}\right)^2+(y+0)^2+(z-2)^2=\tfrac{21}{4}.$$

$(x-a)^2+(y-b)^2+(z-c)^2=r^2$ sphere ctr. at (a,b,c), rad. r

For us, $r^2=\frac{21}{4}$, so $r=\sqrt{\frac{21}{4}}=\frac{\sqrt{21}}{2}$,

and the center is $\left(-\frac{3}{2},0,2\right)$.

EXAMPLES 2

(a) $\{(x,y,z): x^2+y^2+z^2<4\}$

(b) $\{(x,y,z): x^2+y^2+z^2\leq 4\}$

(c) $\{(x,y,z): x^2+y^2+z^2>4\}$

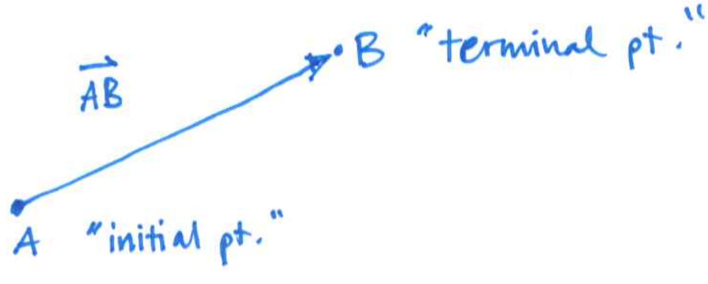
(d) $\{(x,y,z): x^2+y^2+z^2=4,\ z\leq 0\}$

Vectors.

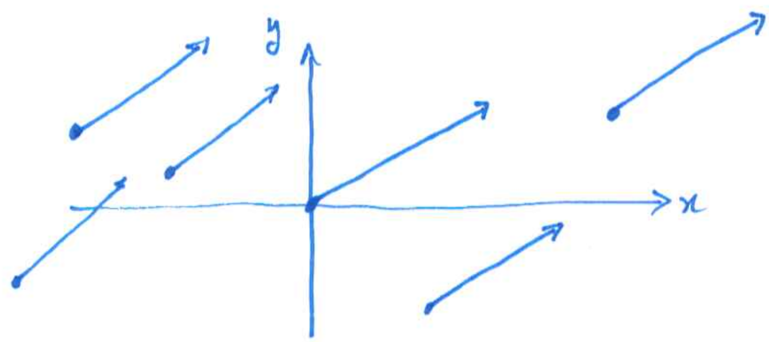
Describe qties. w/ both magnitude and direction, e.g.

- Force
- Displacement
- Velocity

In $\mathbb{R}^3$, a vector is graphed as a directed line segment:



Two vectors in $\mathbb{R}^3$ are "equal" if they have the same magnitude & direc'n, regardless of the initial pt.



Typically, the vector w. initial pt. at the origin is the one used in computations to represent all equal vectors, & is said t/b in "standard form".

When working with standard vectors, we represent them by giving the (cartesian) coordinates of the terminal points:

<u>DEF</u>. If $\vec{v}$ is a vector in $\mathbb{R}^3$ whose std. form has terminus $P(v_1, v_2, v_3)$, then the <u>component form</u> of $\vec{v}$ is given as

$$\vec{v} := \langle v_1, v_2, v_3 \rangle$$