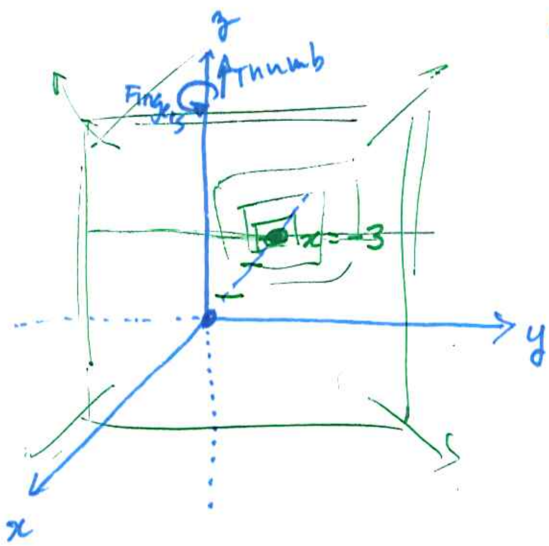
Nov. 18: 3D Coordinate Systems; Vectors.



"Right-handed Cartesian coordinate system"

Points are labelled $P(x, y, z)$.

Origin at $(0,0,0)$

Planes determined by the coordinate axes:

- xy-plane $(z=0)$
- yz-plane $(x=0)$
- xz-plane $(y=0)$

These planes divide 3D space into 8 <u>octants</u>, similar to 2D quadrants. The "first octant" has all coords (x, y, z) positive — there is no convention for numbering the other octants.

<u>Ex. ①</u> (a) $z \geq 0$ is the "<u>half-space</u>" consisting of the points on & above the xy-plane. Points (x, y, z) have $z \geq 0$.

(b) $x = -3$ is the <u>plane</u> perpendicular $(\perp)$ to the x-axis (or parallel $(\parallel)$ to the yz-plane) at $x = -3$. Points $(-3, y, z)$.

(c) $\{z = 0, x \leq 0, y \geq 0\}$ is the 2^{nd} <u>quadrant</u> of the xy-plane.

(d) $\{(x,y,z) : x \geq 0, y \geq 0, z \geq 0\}$ is the 1^{st} <u>octant</u> of 3D space.

(e) $\{(x,y,z) : -1 \leq y \leq 1\}$ is the <u>slab</u> btwn. (incl.) planes $y = -1$ & $y = 1$.

(f) $\{(x,y,z) : y = -2, z = 2\}$ is the <u>line</u> in which the planes $y = -2$ & $z = 2$ intersect. Also, the line through $(0, -2, 2)$ parallel to the x-axis.

Ex (2). $\left.\begin{aligned} x^2+y^2 &= 4 \\ z &= 3 \end{aligned}\right\}$ are satisfied simultaneously by which points — what shape?

Circle, ctr. at $(0,0,3)$, w. radius 2, in the plane $z=3$.

Think: $x^2+y^2=4$ is a cylindrical shell in 3D, & it intersects the plane $z=3$ in a circle.

Recall: In 2D, the dist. btwn. (x_1, y_1) and (x_2, y_2) is:

$$\text{dist} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

In 3D, the dist. btwn. (x_1, y_1, z_1) and (x_2, y_2, z_2) is:

$$\text{dist} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

(can prove this using the Pythagorean thm.)

Ex. 3 compute dist. btwn. $(2, 1, 5)$ & $(-2, 3, 0)$.

$$\begin{aligned} \text{dist} &= \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2} \\ &= \sqrt{(-4)^2 + (2)^2 + (-5)^2} \\ &= \sqrt{16+4+25} \\ &= \sqrt{45} = \sqrt{9\cdot 5} = \sqrt{9}\cdot\sqrt{5} = 3\sqrt{5} \approx \underline{\qquad} \end{aligned}$$