Lec. 25: Nov. 2, 2016.

Housekeeping: HW 5 due today in class

Canvas HW due Friday.

Last time: Surfaces of revol'n

Polar coord.

This time: Polar coord.

Remember (?) :
$$\begin{cases} x = r\cos(\theta) \\ y = r\sin(\theta) \end{cases}$$
parametrized "unit" circle

$$x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta$$
$$= r^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1}$$

$$\boxed{x^2 + y^2 = r^2}$$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta,$$

so $\boxed{\frac{y}{x} = \tan\theta}$

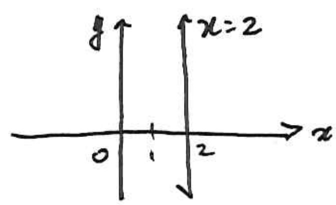
These two rules give a way of relabelling polar coordinates as cartesian & vice-versa.

They give a way of translating relationships btwn. r & θ into relationships btwn. x & y.

Example ① Polar: $r\cos\theta = 2$

Cartesian: $\boxed{x = 2}$

$$r\cos\theta = x$$
$$r\sin\theta = y$$
$$\frac{x}{y} = \tan\theta$$
$$x^2+y^2 = r^2$$



Example ② Cartesian: $xy = 4$

Polar: $(r\cos\theta)(r\sin\theta) = 4$

$$r^2\sin\theta\cos\theta = 4$$

$$\boxed{r^2 = \frac{4}{\sin\theta\cos\theta} = 4\csc\theta\sec\theta}$$

Example ③ Polar: $r = 1 + 2r\cos\theta$

$$\frac{r-1}{2} = r\cos\theta$$

$$x = \frac{r-1}{2} = \frac{\pm\sqrt{x^2+y^2} - 1}{2}$$

$$\pm\sqrt{x^2+y^2} = 2x+1$$

$$x^2+y^2 = (2x+1)^2 = 4x^2+4x+1$$

$$y^2 = 3x^2+4x+1$$

Example (4) Polar: $r = 1 - \cos\theta$

Example (5) Find a polar eq'n for the circle of rad. 3, centered at $(0,3)$.

Cartesian: $(x-0)^2 + (y-3)^2 = 3^2$

$$x^2 + (y-3)^2 = 9$$