

Homework 8: Due in class December 07

Reminder

Your submitted homework solutions should show not only your answers, but should show a clearly reasoned logical argument, written using **complete English sentences**, leading to that solution. Each mathematical symbol that you will encounter stands for one or more English words¹, and if you elect to use symbols, you must use them properly. In particular, please avoid the use of the “running equals sign”, as this is an abuse of notation and is unacceptable: http://www.wikiwand.com/en/Equals_sign#/Incorrect_usage. Write your solutions so that a student one course behind you in the sequence would understand them.

Problem 1. [2 points] What is the difference between $(1, 2)$ and $\langle 1, 2 \rangle$?

Problem 2. [2 points] What effect does multiplying a vector by the scalar -2 have?

Problem 3. [6 points] For each of the following pairs of points P and Q , give the standard form of the vector \vec{PQ} , compute its magnitude, and give the standard form of a unit vector in the same direction as \vec{PQ} .

(a) $P := (2, 1, 2)$, $Q := (4, 3, 2)$

(b) $P := (3, 2)$, $Q := (7, -2)$

(c) $P := (0, 3, -1)$, $Q := (6, 2, 5)$

Problem 4. [10 points] This problem is about the famous Cauchy-Schwarz inequality. Here, we will prove this inequality in \mathbb{R}^3 , but it holds in any vector space².

(a) Since $\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$, where θ is the angle between \vec{u} and \vec{v} , show that the inequality $|\vec{u} \cdot \vec{v}| \leq |\vec{u}| |\vec{v}|$ holds for any vectors \vec{u} and \vec{v} .

(b) Under what circumstances, if any, does $|\vec{u} \cdot \vec{v}|$ equal $|\vec{u}| |\vec{v}|$? You must justify your answer.

¹See a list of mathematical symbols and their meanings here: http://en.wikipedia.org/wiki/List_of_mathematical_symbols

²Read more about the inequality here: https://en.wikipedia.org/wiki/Cauchy%E2%80%93Schwarz_inequality .