Lec. 26: Nov. 4, 2016,

Housekeeping: • Written HW due Weds. - day of exam

• No canvas HW

• Exam 2 on Wednesday

Last Time: • Polar coord.

This time: • Polar coord. practice.

Polar coords.

Example ① Find a corresponding cartesian eq'n

$$\left.\begin{aligned} x &= r\cos\theta \\ y &= r\sin\theta \\ x^2+y^2 &= r^2 \\ \frac{y}{x} &= \tan\theta \end{aligned}\right\}$$

$$r\cos\theta = -4$$

$$x = -4 \quad \text{(vertical line)}$$

Example ② $$r^2 = 4r\cos\theta$$

$$x^2+y^2 = 4x$$

$$x^2 - 4x + y^2 = 0$$

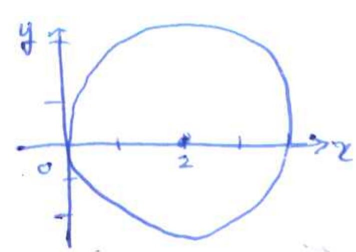
Aside: Completing the square.

$$x^2 - 4x = \underbrace{x^2 - 4x + 4}\ -4$$
$$= (x-2)^2 - 4$$

$$x^2 - 4x + y^2 = 0 \iff (x-2)^2 + y^2 = 4$$

Circle of radius 2, ctr. at $(2,0)$

$$\boxed{r^2 = 4r\cos\theta}$$



Example ③ $r = \dfrac{4}{2\cos\theta - \sin\theta}$. Graph the eq'n.

Graphing eq'ns given in Polar coordinates.

Tricks for developing intuition:

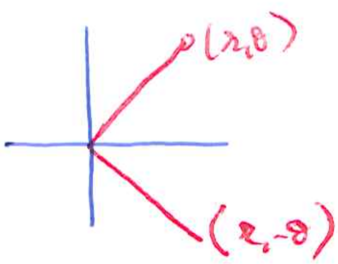
- SYMMETRY

(1) About x-axis: $\underset{\text{on graph}}{(r,\theta)} \Rightarrow \underbrace{(r,-\theta) \text{ or } (-r,\pi-\theta)}_{\text{are also on the graph}}$

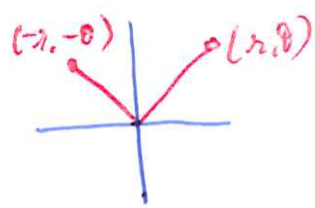
(2) Abt. y-axis: $\underset{\text{on graph}}{(r,\theta)} \Rightarrow \underset{\text{on graph}}{(-r,-\theta)}$ or $\underset{\text{on graph.}}{(r,\pi-\theta)}$

(3) Abt. origin: $\underset{\text{on graph}}{(r,-\theta)} \Rightarrow \underbrace{(-r,\theta) \text{ or } (r,\theta+\pi)}_{\text{on graph}}$

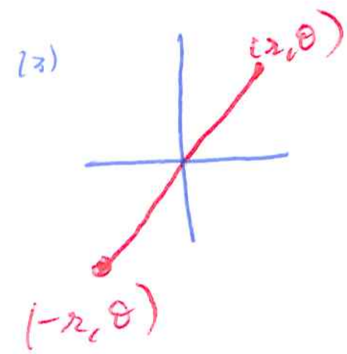
(1)



(2)



(3)



More tricks for developing intuition

- **SLOPE.**

If the polar eq'n is $r = f(\theta)$, and $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Then:

$$\boxed{\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta},}$$

$\left(\text{provided } \left.\frac{dx}{d\theta}\right|_{(r,\theta)} \neq 0. \right)$

Suppose $r = f(\theta)$ passes through the origin when $\theta = \theta_0$.

Then $f(\theta_0) = 0$, so $\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \frac{\cancel{f'(\theta_0)}\sin(\theta_0)}{\cancel{f'(\theta_0)}\cos(\theta_0)} = \tan\theta_0.$

If you know that the graph of a polar eq'n crosses origin at $\theta = \theta_0$, then the slope of the line tangent to the graph at the origin is $\tan\theta_0$.

- **PERIODICITY.**

$x = r\cos\theta$
$y = r\sin\theta$.

If we're given $r = f(\theta)$, then $\begin{cases} x = f(\theta)\cos\theta \\ y = f(\theta)\sin\theta. \end{cases}$

For some functions $f(\theta)$, can exploit periodicity to restrict the range of θ-values we check / plot.

- TABLE OF VALUES. – like for other parametric curves.

$x = f(\theta)\cos\theta$

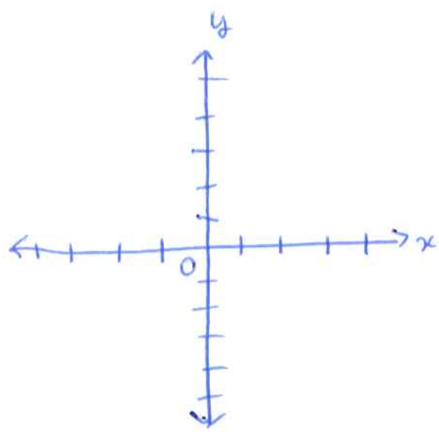
$y = f(\theta)\sin\theta$

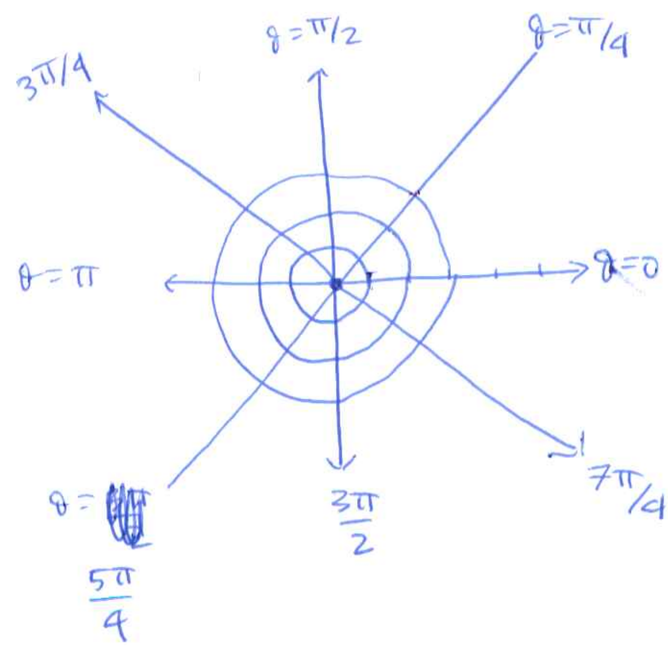
	$x = f(\theta)\cos\theta$	$y = f(\theta)\sin\theta$	(x, y)
$\theta = 0$			
$\theta = \pi/4$			
$\theta = \pi/2$			
$\vdots$			

OR

$r = f(\theta)$

	$r = f(\theta)$	(r, θ)
$\theta = 0$		
$\theta = \pi/4$		
$\pi/2$		
$\vdots$		





Example ① Graph $r = 1 - \cos\theta$ in the coord. plane.

- SYMMETRY? If (r, θ) is on the graph, then

$$r = 1 - \cos\theta \quad \text{is satisfied.}$$

Since $\cos(-\theta) = \cos(\theta)$ (cos is an even fn.),

this means tht. also, $r = 1 - \cos(-\theta)$, so

$(r, -\theta)$ should be on the graph. Symmetry abt. x-axis.

- SLOPE AT ORIGIN?

$r = 0$ when $0 = 1 - \cos\theta$, i.e., $\cos\theta = 1$.

So $r = 0$ when $\theta_0 = 0 + 2m\pi$, for $m \in \mathbb{Z}$. At the origin, the slope is

$$\left.\frac{dy}{dx}\right|_{(0, 2\pi m)} = \tan(\theta_0) = \tan(2\pi m)$$

$$= \frac{\sin(2\pi m)}{\cos(2\pi m)} = 0.$$

The ~~slope~~ tangent line at the origin is horizontal.

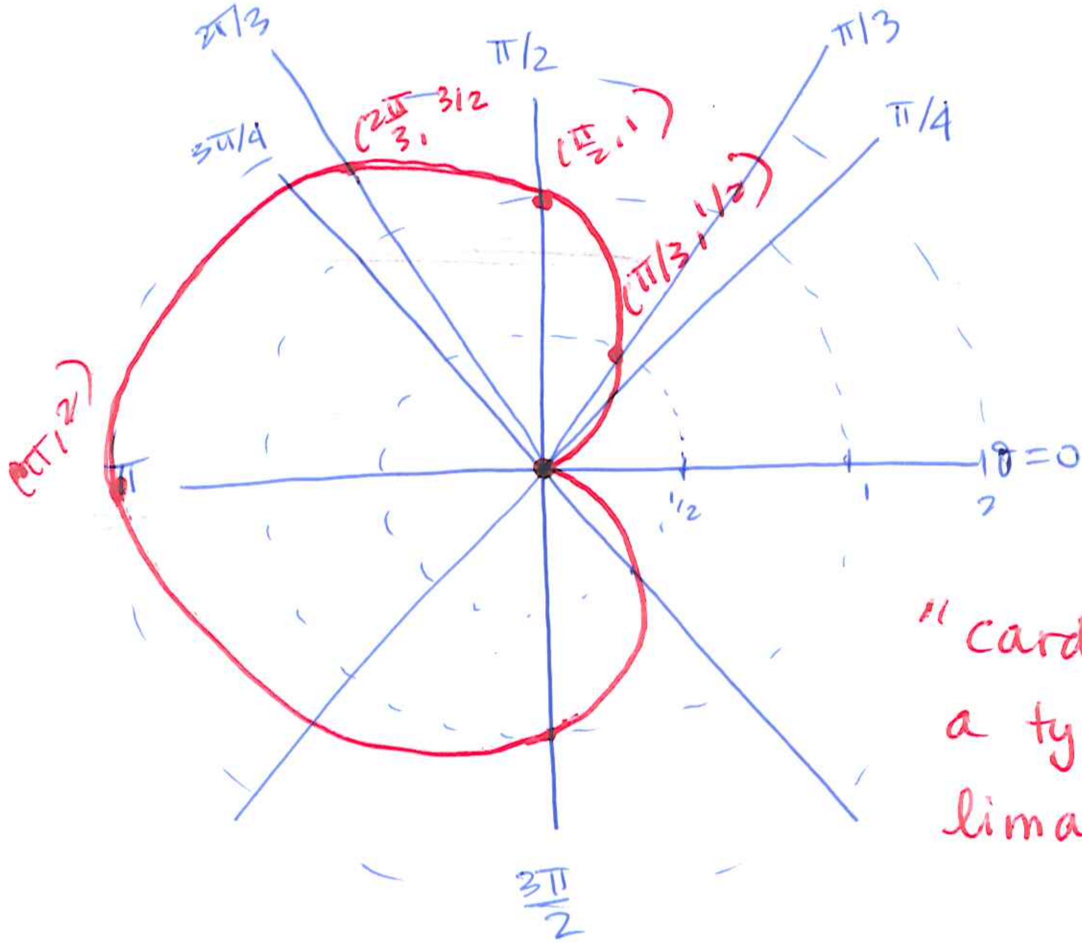
- PERIODICITY.

$1 - \cos\theta$ is 2π-periodic, since $1 - \cos(\theta + 2\pi) = 1 - \cos(\theta)$

So when choosing θ-vals for the table, can restrict to $[0, 2\pi]$, and further (symmetry), to $[0, \pi]$.

Ex. ct'd.

θ	0	$\pi/4$	$\pi/3$	$\pi/2$	$2\pi/3$	π
$r = 1-\cos\theta$	0	$\frac{2-\sqrt{2}}{2}$	$1/2$	1	$3/2$	2



"cardioid" — a type of limaçon