NOV. 28 : Vectors, ct'd.

Last time : vectors in the plane
canonical form / standard form.

EXERCISE 1 Find the component form of the vector $\overrightarrow{PQ}$, where $P(-3,4,1)$ and $Q(-5,2,2)$.

$$\vec{v} := Q-P = \langle -5-(-3),\ 2-4,\ 2-1 \rangle$$
$$= \langle -2, -2, 1 \rangle.$$

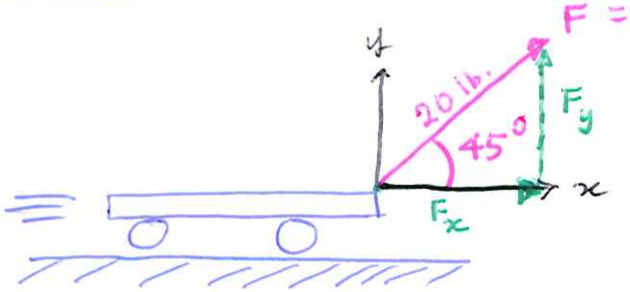
DEF. The magnitude or length of the vector $\vec{v} = \overrightarrow{PQ}$ $\left(\vec{v} = \langle v_1, v_2, v_3 \rangle,\ P(x_1, y_1, z_1),\ Q(x_2, y_2, z_2)\right)$ is :

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \underbrace{\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}}_{\text{dist. formula in 3D}}$$

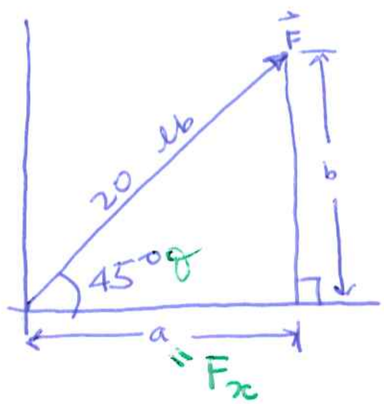
EX. 2 Find $|\vec{v}|$ where $\vec{v} := PQ$, $P(-3,4,1)$, $Q(-5,2,2)$.

$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{4+4+1} = \sqrt{9}$$
$$= 3.$$

EX: 3



Cart is being pulled along a smooth horizontal floor with a 20-lb. force $\vec{F}$ making a 45° angle to the floor. what's the effective force moving the cart forward?



The effective force moving the cart forward is just the x-component of $\vec{F}$.

For the right triangle,

$$\sin\theta = \frac{\text{OPPOSITE}}{\text{HYPOTENUSE}}, \quad \cos\theta = \frac{\text{ADJ.}}{\text{HYP.}}, \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\text{OPP.}}{\text{ADJ.}}$$

$S\frac{O}{H}\ C\frac{A}{H}\ T\frac{O}{A}$

$$\cos(45°) = \frac{F_x}{20\text{ lb.}} \Rightarrow F_x = (20\text{ lb})\cos(45°)$$
$$= (20\text{ lb})\left(\frac{\sqrt{2}}{2}\right)$$
$$= 10\sqrt{2}\text{ lb}$$
$$\approx 14.14\text{ lb.}$$

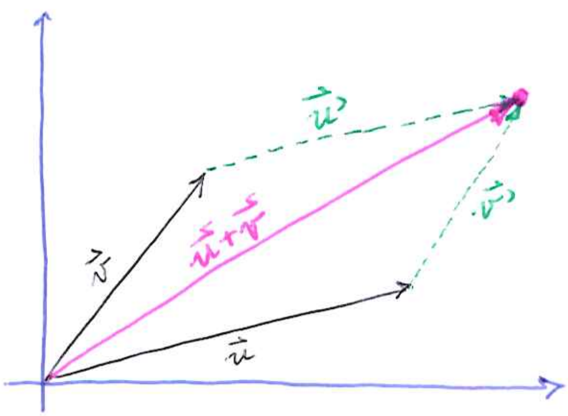
Vector algebra.

DEF. Let $\vec{u} := \langle u_1, u_2, u_3 \rangle$, $\vec{v} := \langle v_1, v_2, v_3 \rangle$, and let $k \in \mathbb{R}$.

Then: $\vec{u} + \vec{v} := \langle u_1 + v_1 , u_2 + v_2 , u_3 + v_3 \rangle$

$k\vec{u} := \langle ku_1 , ku_2 , ku_3 \rangle$

- "Head-to-tail add'n"



$\vec{u} + \vec{v} = \overline{u+v}$

- What is the magnitude/length of $k\vec{u}$?

$$\left.\begin{aligned} \vec{u} &= \langle u_1, u_2, u_3 \rangle \\ ku &= \langle ku_1, ku_2, ku_3 \rangle \end{aligned}\right\} \Rightarrow$$

$$\begin{aligned} |k\vec{u}| &= \sqrt{(ku_1)^2 + (ku_2)^2 + (ku_3)^2} \\ &= \sqrt{k^2u_1^2 + k^2u_2^2 + k^2u_3^2} \\ &= \sqrt{k^2(u_1^2 + u_2^2 + u_3^2)} \\ &= \sqrt{k^2}\sqrt{u_1^2 + u_2^2 + u_3^2} \\ &= |k| \cdot |\vec{u}| \end{aligned}$$

→ this $|\cdot|$ is vector length

→ this $|\cdot|$ is abs. val. (of scalar)

Ex. 4 Let $\vec{u} := \langle -1, 3, 1 \rangle$, $\vec{v} := \langle 4, 7, 0 \rangle$. Find:

(a) $$2\vec{u} + 3\vec{v} = \langle 2(-1) + 3(4),\ 2(3) + 3(7),\ 2(1) + 3(0) \rangle$$
$$= \langle -2 + 12,\ 6 + 21,\ 2 + 0 \rangle$$
$$= \langle 10, 27, 2 \rangle$$

(b) $$\vec{u} - \vec{v} = \langle -1 - 4,\ 3 - 7,\ 1 - 0 \rangle$$
$$= \langle -5, -4, 1 \rangle$$

(c) $$\left| \tfrac{1}{2}\vec{u} \right| = \left| \tfrac{1}{2} \right| \cdot |\vec{u}| = \tfrac{1}{2}\sqrt{(-1)^2 + (3)^2 + (1)^2}$$
$$= \tfrac{1}{2}\sqrt{1 + 9 + 1}$$
$$= \tfrac{1}{2}\sqrt{11}.$$